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KWG presents

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第10回 電気数学
三角関数(2)

2022.11.05 Sat

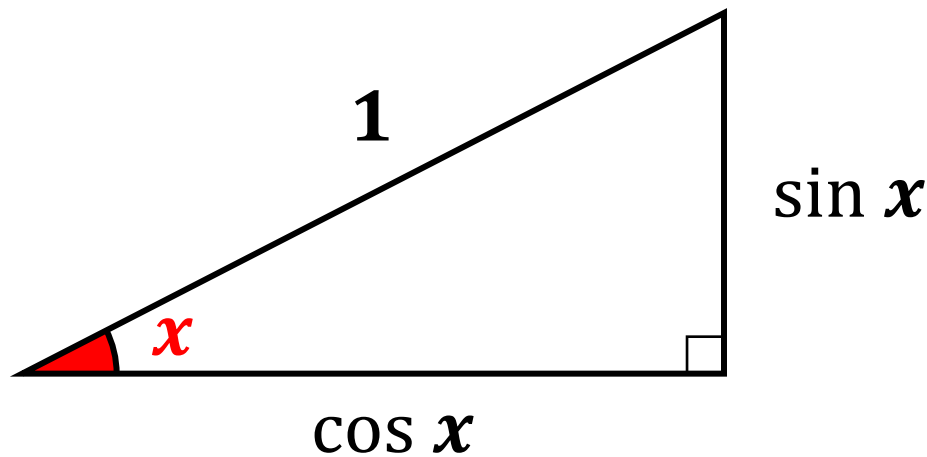
三角関数

直角三角形の1つの角を変数 x で表し、 x に対する三角比の値を y とした関数を“三角関数”という

$$y = \sin x$$

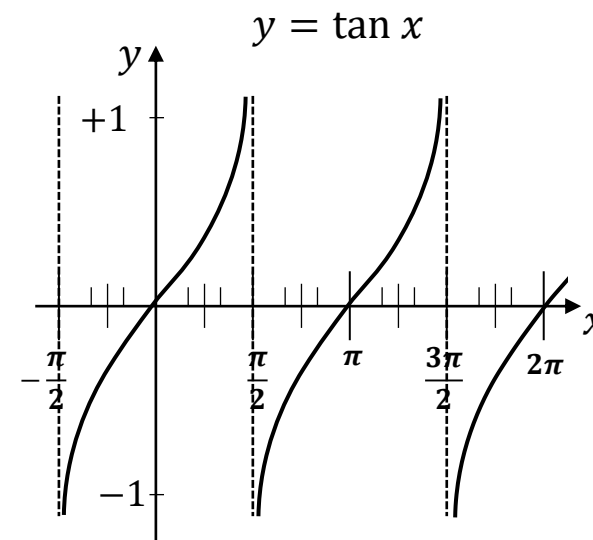
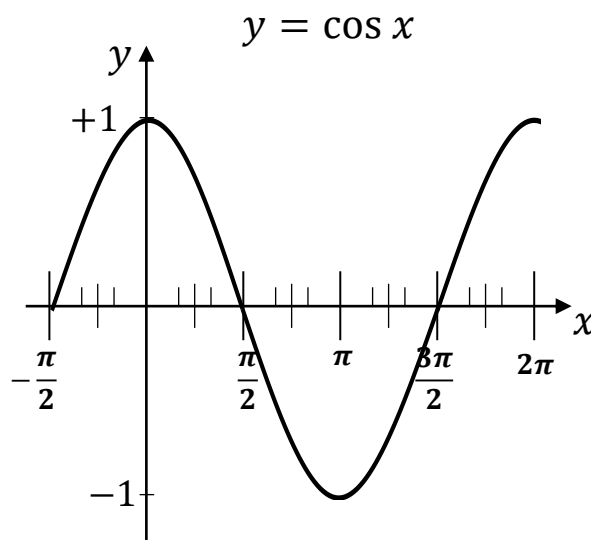
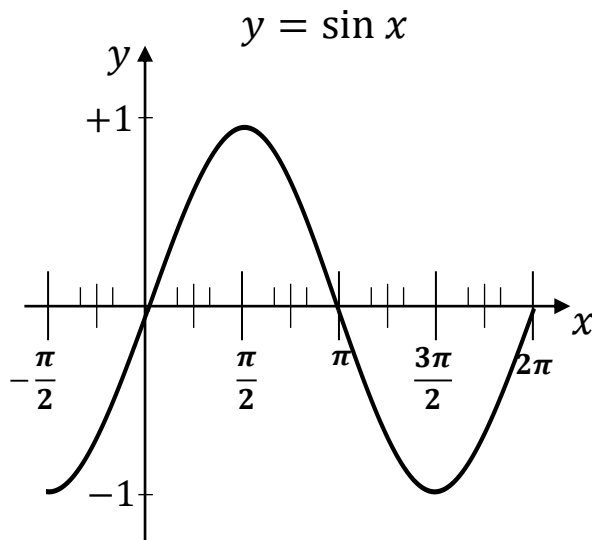
$$y = \cos x$$

$$y = \tan x$$



x [rad]	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$y = \sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
$y = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$y = \tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	0

三角関数

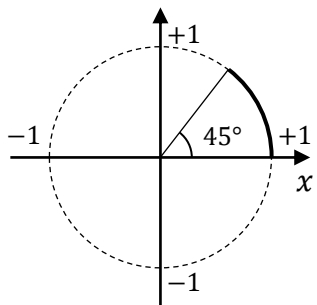


x [rad]	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$y = \sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0
$y = \cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$y = \tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	/	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	0

練習問題 I

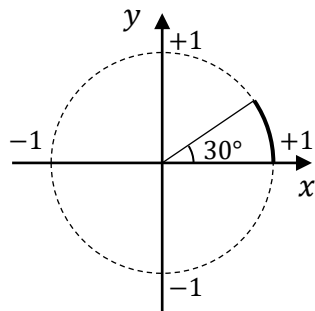
各問に答えよ。各問の円は半径 1 の単位円である。

(1)



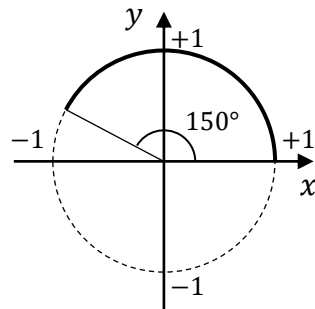
Ans. $\cos 45^\circ =$ _____

(2)



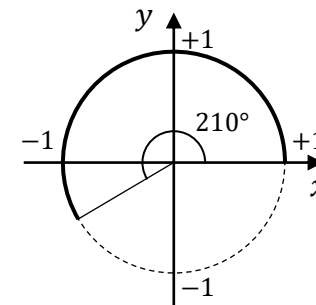
Ans. $\cos 30^\circ =$ _____

(3)



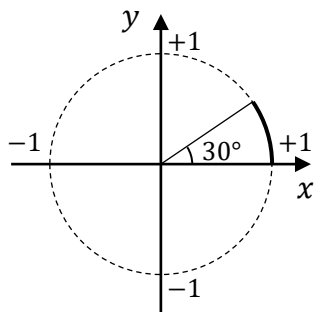
Ans. $\cos 150^\circ =$ _____

(4)



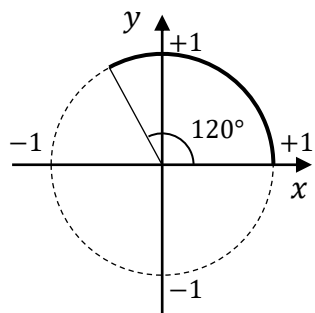
Ans. $\cos 210^\circ =$ _____

(5)



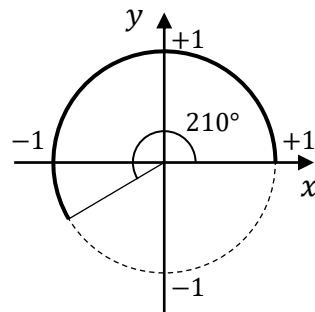
Ans. $\sin 30^\circ =$ _____

(6)



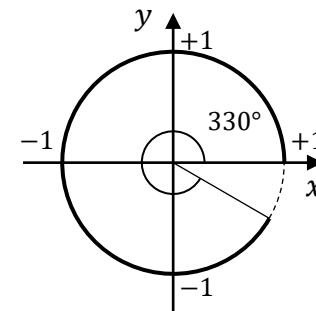
Ans. $\sin 120^\circ =$ _____

(7)



Ans. $\sin 210^\circ =$ _____

(8)

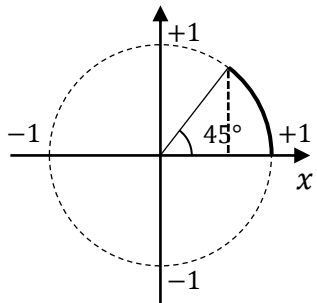


Ans. $\sin 330^\circ =$ _____

練習問題 I (解答)

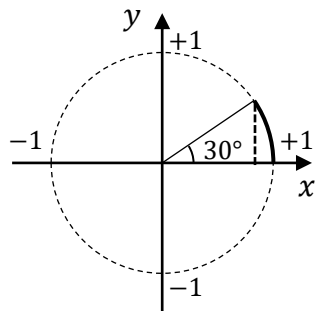
各問に答えよ。各問の円は半径 1 の単位円である。

(1)



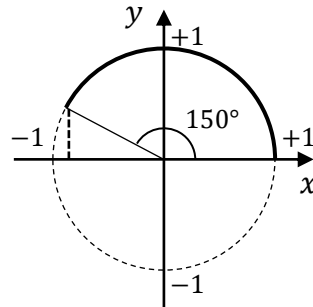
Ans. $\cos 45^\circ = \frac{1}{\sqrt{2}}$

(2)



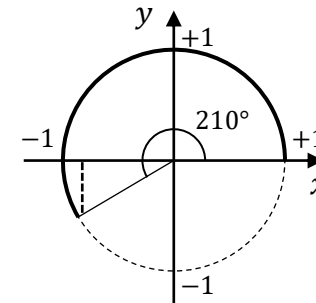
Ans. $\cos 30^\circ = \frac{\sqrt{3}}{2}$

(3)



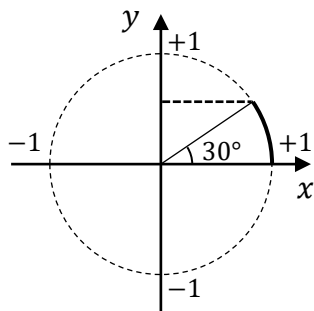
Ans. $\cos 150^\circ = -\frac{\sqrt{3}}{2}$

(4)



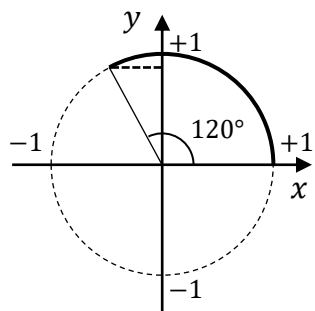
Ans. $\cos 210^\circ = -\frac{\sqrt{3}}{2}$

(5)



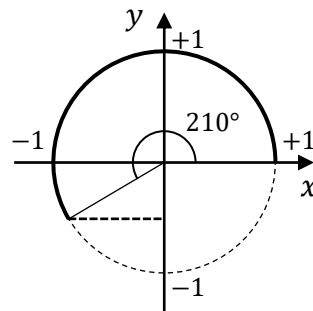
Ans. $\sin 30^\circ = \frac{1}{2}$

(6)



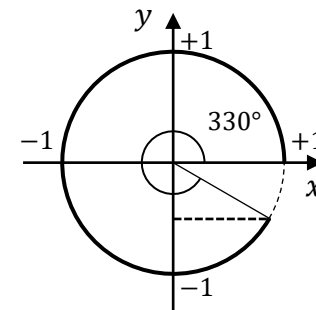
Ans. $\sin 120^\circ = \frac{\sqrt{3}}{2}$

(7)



Ans. $\sin 210^\circ = -\frac{1}{2}$

(8)

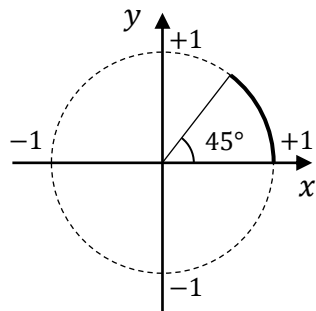


Ans. $\sin 330^\circ = -\frac{1}{2}$

練習問題2

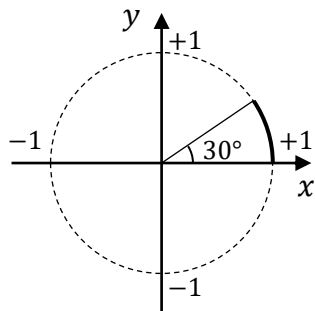
各問に答えよ。各問の円は半径1の単位円である。

(1)



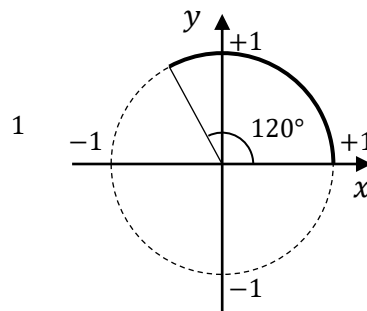
Ans. $\tan 45^\circ =$ _____

(2)



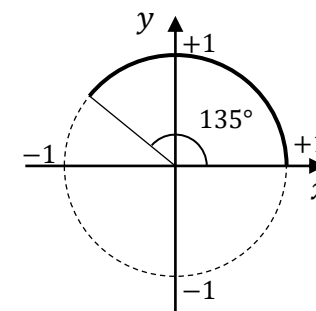
Ans. $\tan 30^\circ =$ _____

(3)



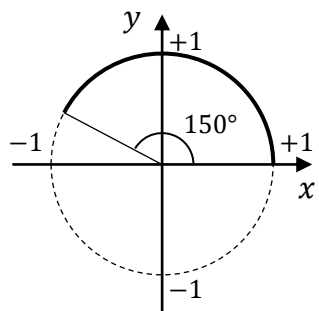
Ans. $\tan 120^\circ =$ _____

(4)



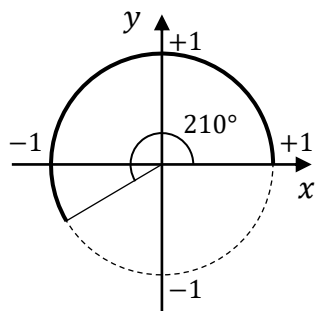
Ans. $\tan 135^\circ =$ _____

(5)



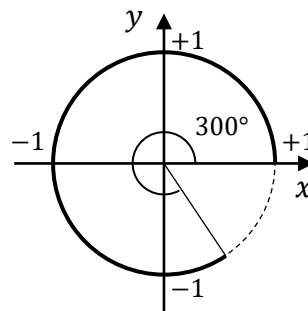
Ans. $\tan 150^\circ =$ _____

(6)



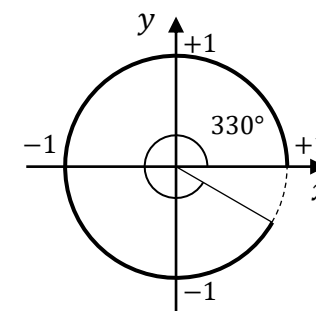
Ans. $\tan 210^\circ =$ _____

(7)



Ans. $\tan 300^\circ =$ _____

(8)

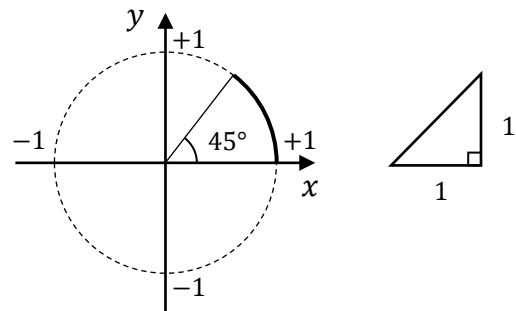


Ans. $\tan 330^\circ =$ _____

練習問題2 (解答)

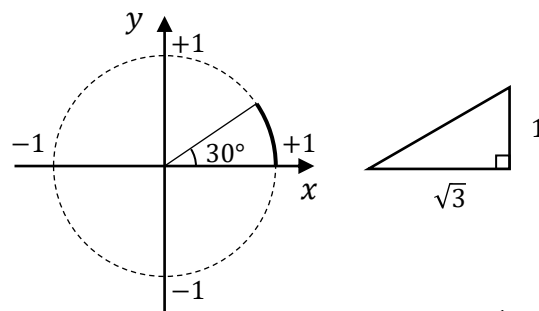
各問に答えよ。各問の円は半径1の単位円である。

(1)



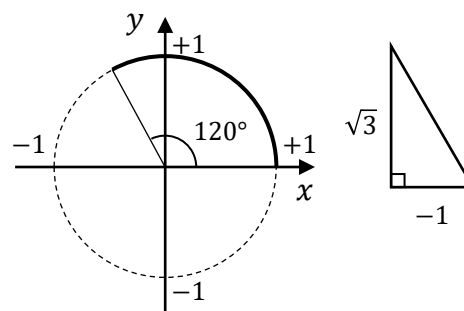
Ans. $\tan 45^\circ = 1$

(2)



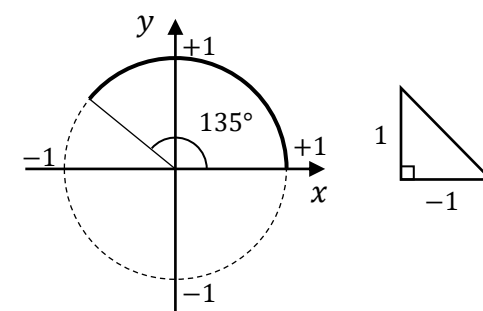
Ans. $\tan 30^\circ = \frac{1}{\sqrt{3}}$

(3)



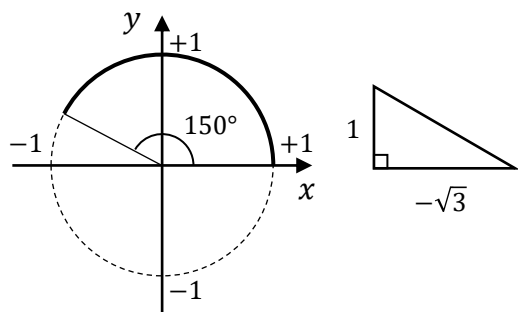
Ans. $\tan 120^\circ = -\sqrt{3}$

(4)



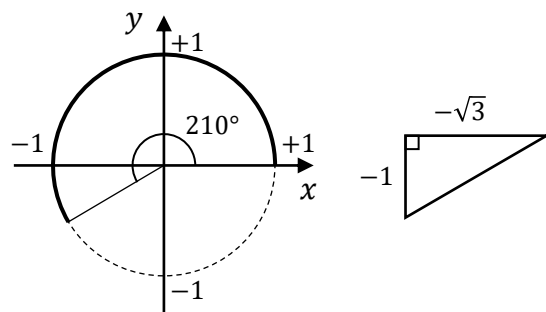
Ans. $\tan 135^\circ = -1$

(5)



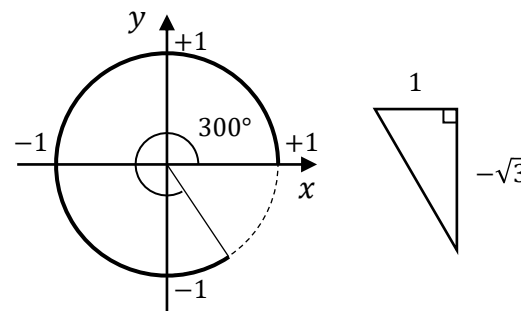
Ans. $\tan 150^\circ = -\frac{1}{\sqrt{3}}$

(6)



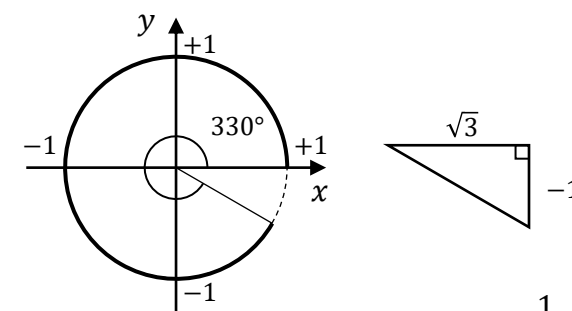
Ans. $\tan 210^\circ = \frac{1}{\sqrt{3}}$

(7)



Ans. $\tan 300^\circ = -\sqrt{3}$

(8)

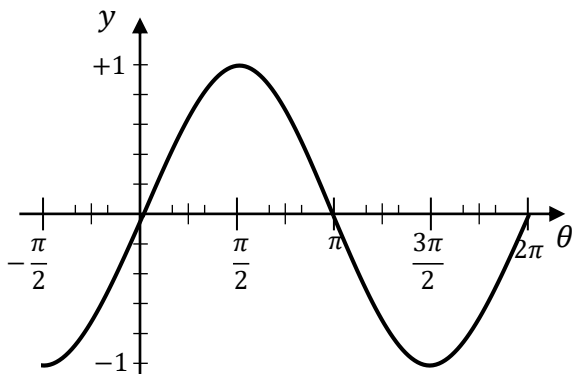


Ans. $\tan 330^\circ = -\frac{1}{\sqrt{3}}$

練習問題3

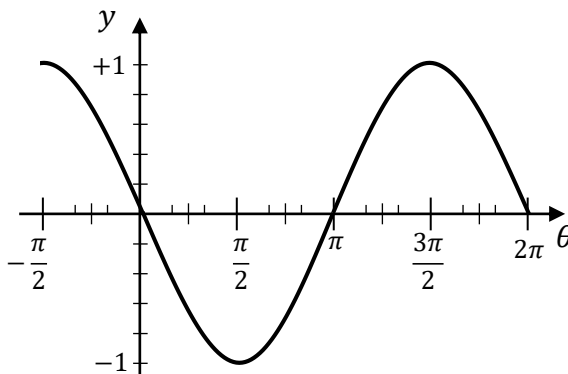
グラフを参照し、各問に答えよ。

(1) $y = 1$ となる角度[rad]示せ



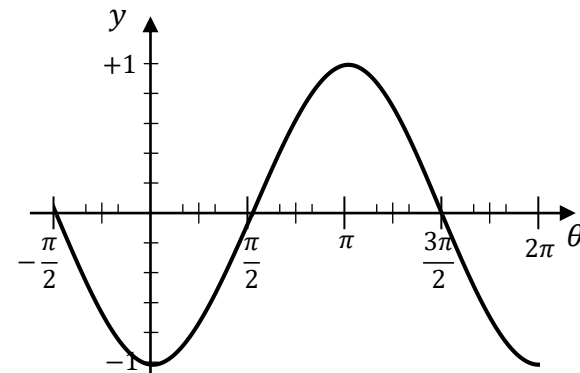
Ans. $\theta =$ _____

(2) $y = 1$ となる角度[rad]示せ



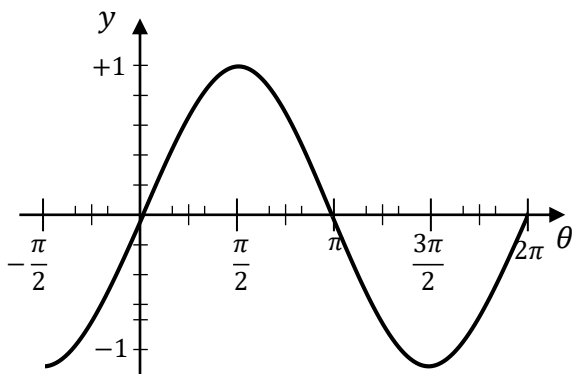
Ans. $\theta =$ _____

(3) $y = 1$ となる角度[rad]示せ



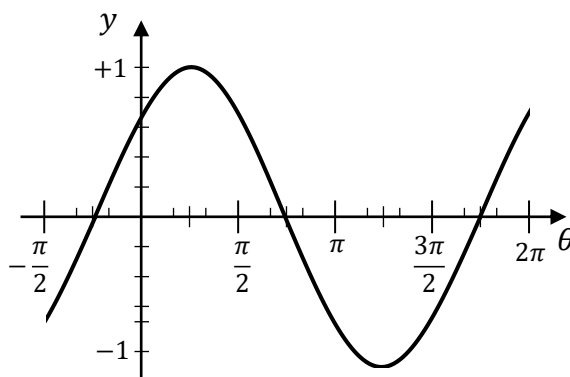
Ans. $\theta =$ _____

(4) $y = 0$ となる角度[rad]示せ



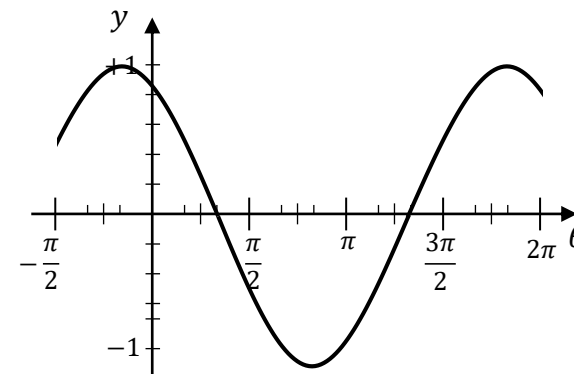
Ans. $\theta =$ _____

(5) $y = 0$ となる角度[rad]示せ



Ans. $\theta =$ _____

(6) $y = 0$ となる角度[rad]示せ

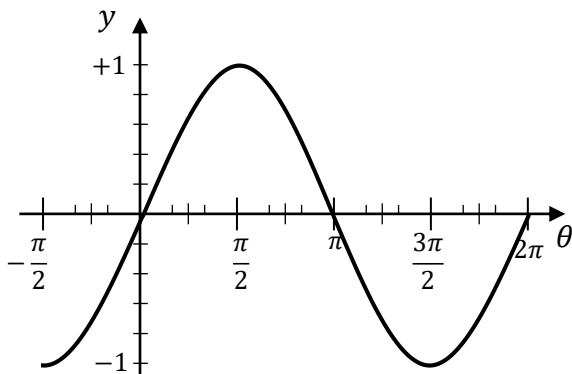


Ans. $\theta =$ _____

練習問題3 (解答)

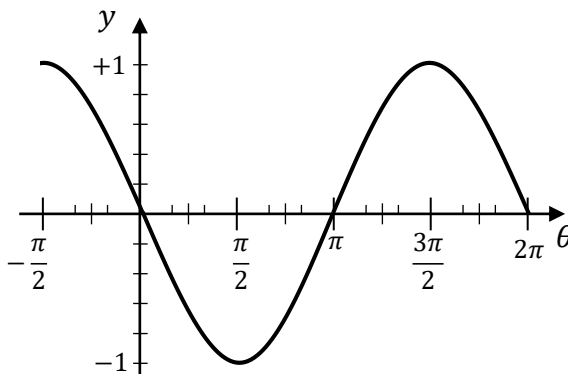
グラフを参照し、各問に答えよ。

(1) $y = 1$ となる角度[rad]示せ



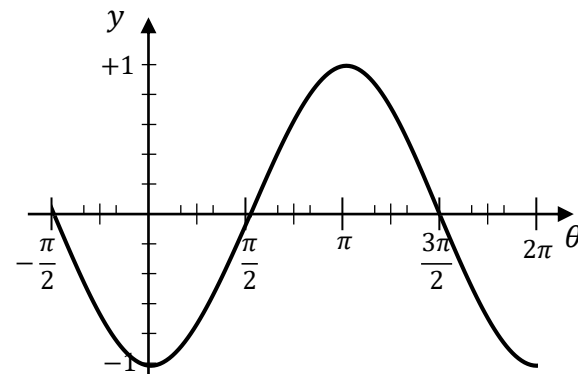
Ans. $\theta = \frac{\pi}{2}$

(2) $y = 1$ となる角度[rad]示せ



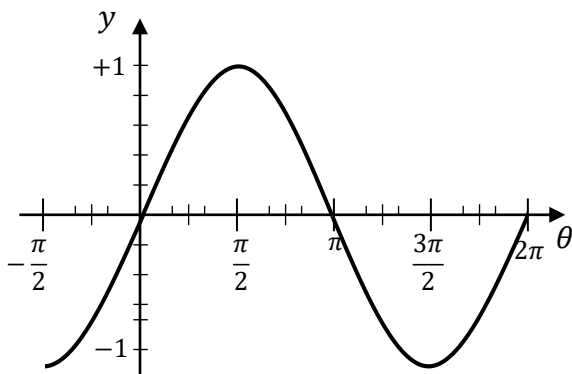
Ans. $\theta = \frac{3\pi}{2}$

(3) $y = 1$ となる角度[rad]示せ



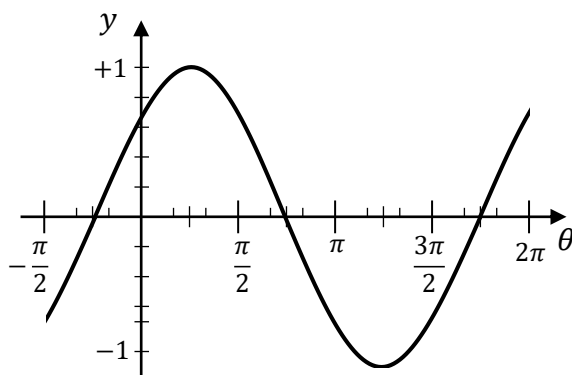
Ans. $\theta = \pi$

(4) $y = 0$ となる角度[rad]示せ



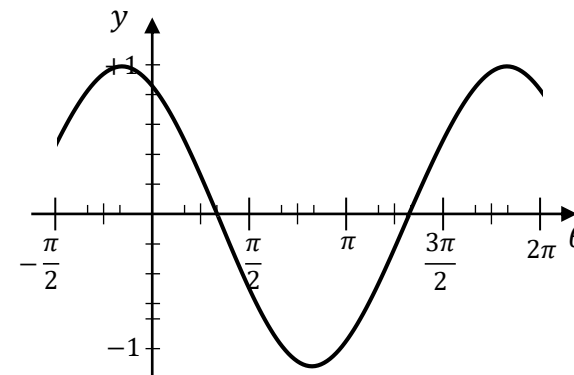
Ans. $\theta = 0, \pi, 2\pi$

(5) $y = 0$ となる角度[rad]示せ



Ans. $\theta = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$

(6) $y = 0$ となる角度[rad]示せ

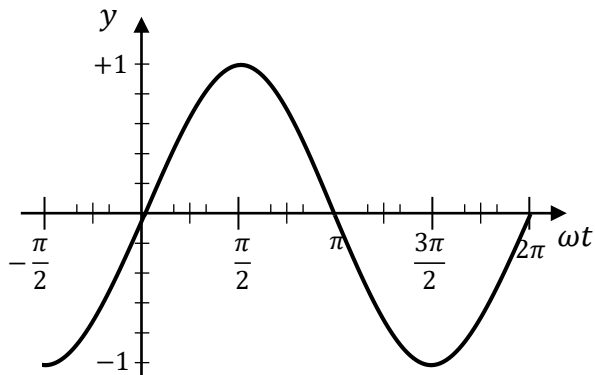


Ans. $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

練習問題4

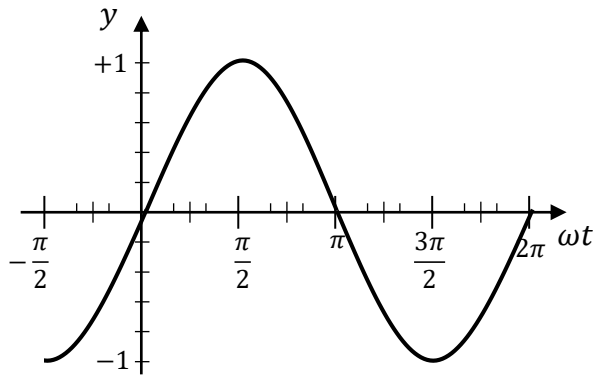
グラフを参照し、各問に答えよ。

(1) $\omega t = \frac{\pi}{2}$ における y の値



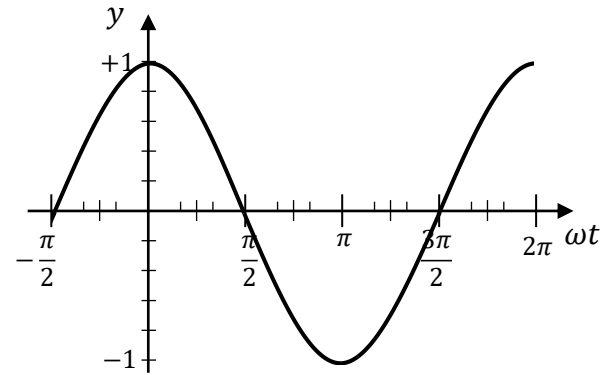
Ans. $y =$ _____

(2) $\omega t = \frac{4\pi}{3}$ における y の値



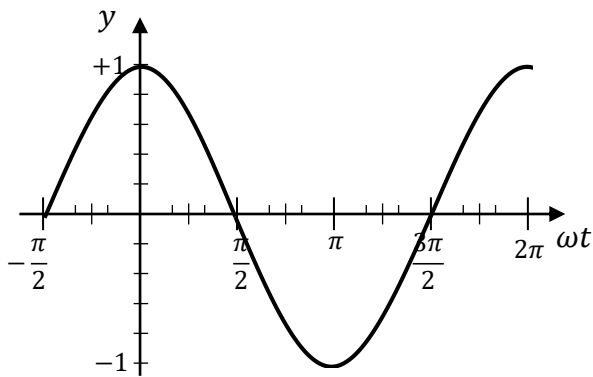
Ans. $y =$ _____

(3) $\omega t = \frac{\pi}{6}$ における y の値



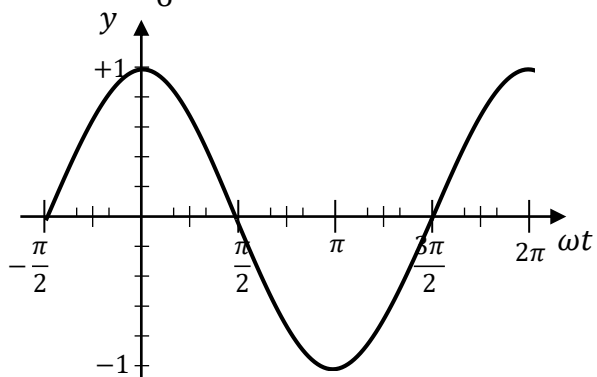
Ans. $y =$ _____

(4) $\omega t = \frac{\pi}{3}$ における y の値



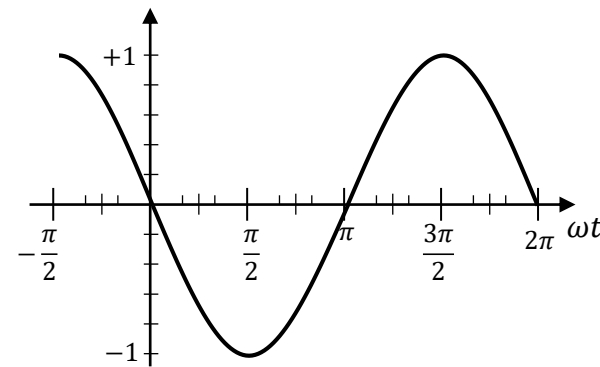
Ans. $y =$ _____

(5) $\omega t = \frac{7\pi}{6}$ における y の値



Ans. $y =$ _____

(6) $\omega t = \frac{3\pi}{4}$ における y の値

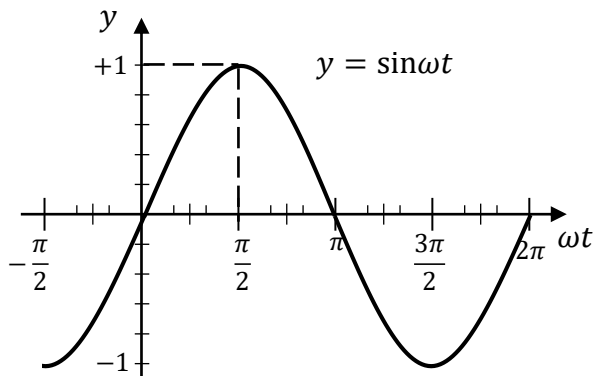


Ans. $y =$ _____

練習問題4 (解答)

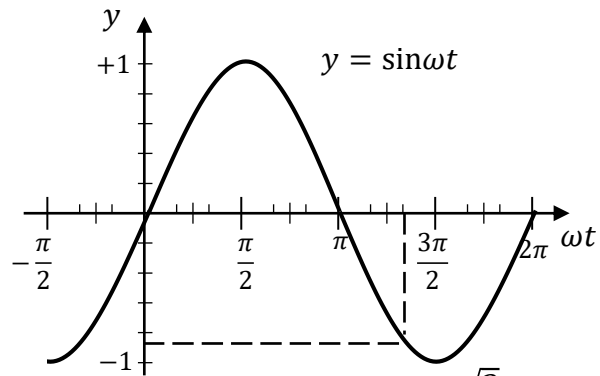
グラフを参照し、各問に答えよ。

(1) $\omega t = \frac{\pi}{2}$ における y の値



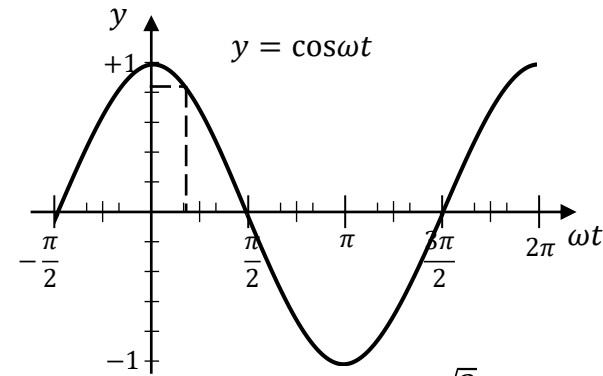
Ans. $y = 1$

(2) $\omega t = \frac{4\pi}{3}$ における y の値



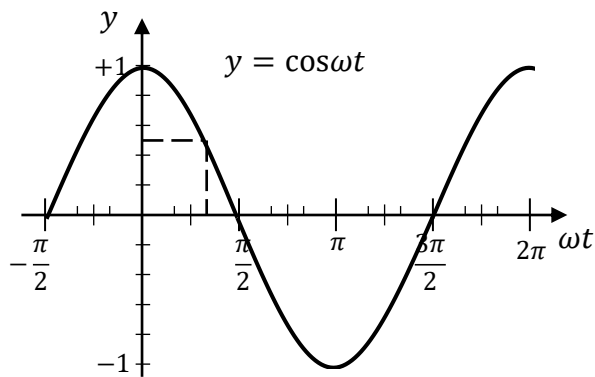
Ans. $y = -\frac{\sqrt{3}}{2}$

(3) $\omega t = \frac{\pi}{6}$ における y の値



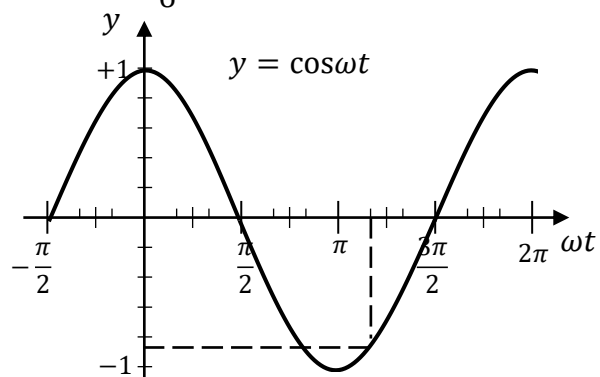
Ans. $y = \frac{\sqrt{3}}{2}$

(4) $\omega t = \frac{\pi}{3}$ における y の値



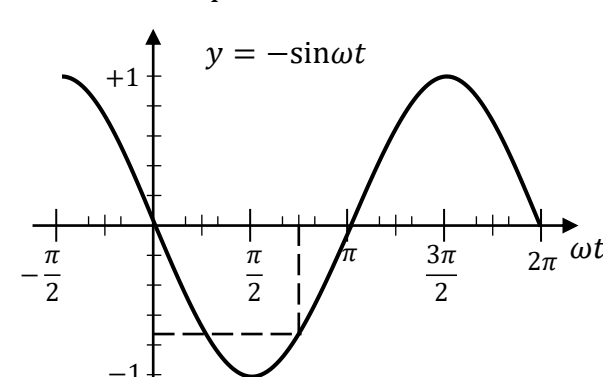
Ans. $y = \frac{1}{2}$

(5) $\omega t = \frac{7\pi}{6}$ における y の値



Ans. $y = -\frac{\sqrt{3}}{2}$

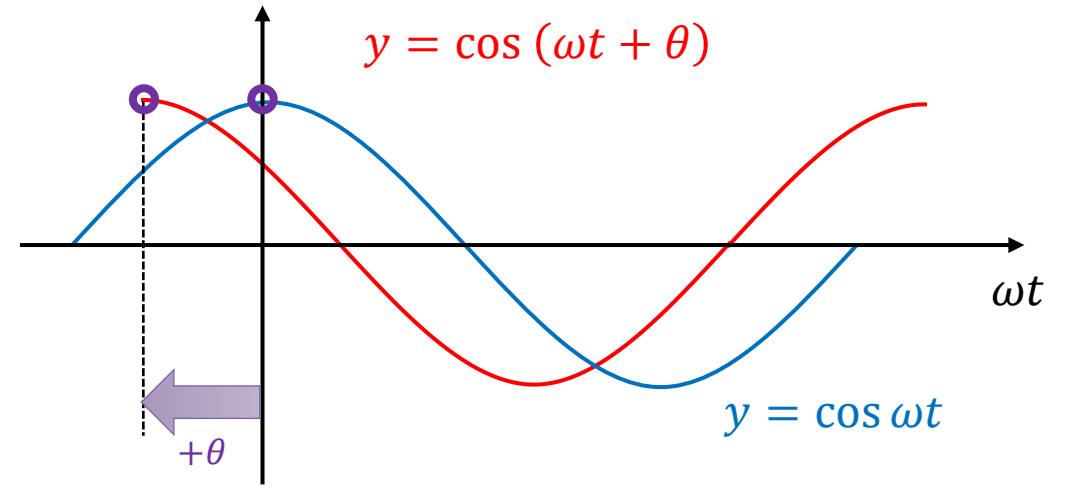
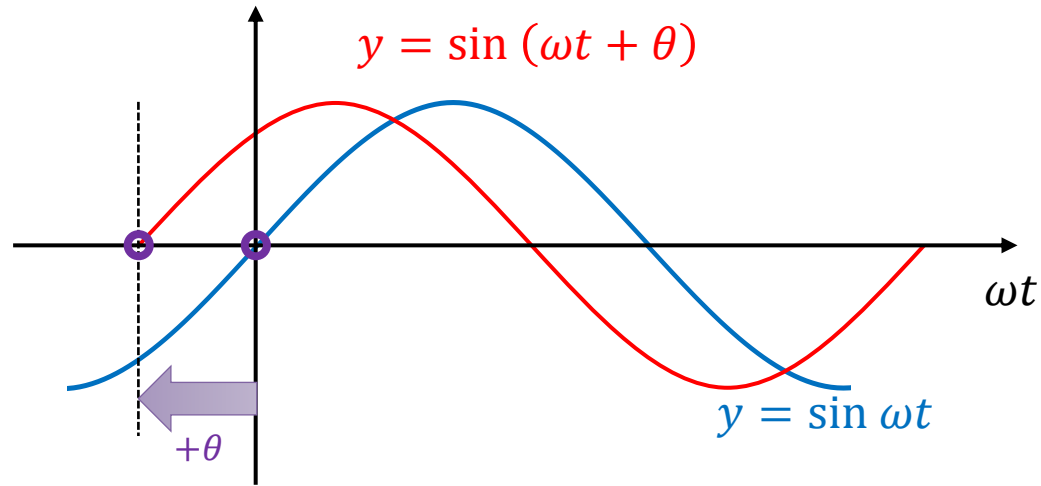
(6) $\omega t = \frac{3\pi}{4}$ における y の値



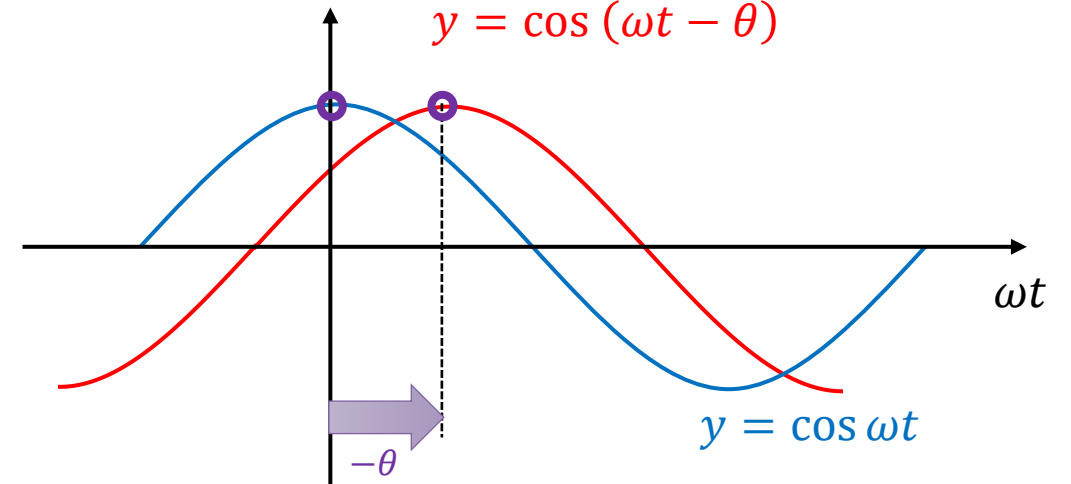
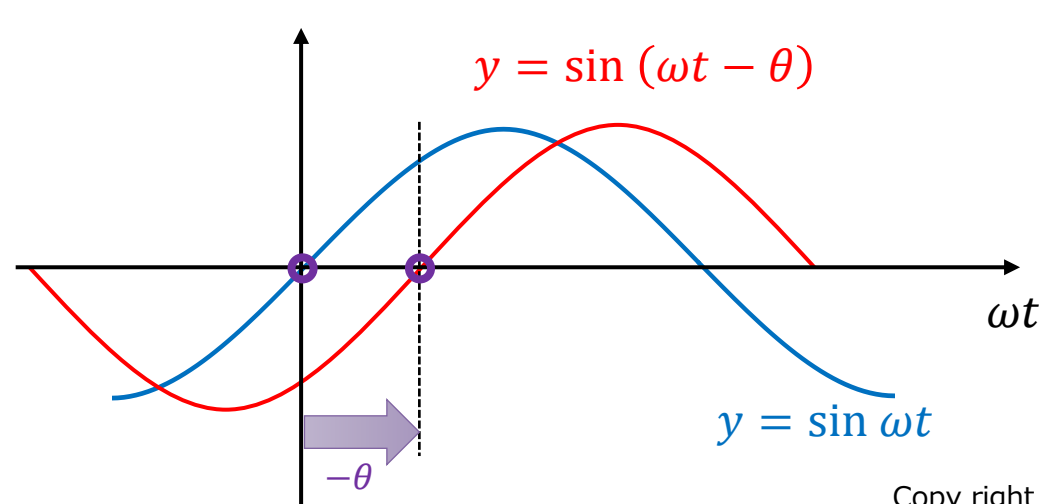
Ans. $y = -\frac{1}{\sqrt{2}}$

波形と位相差

位相差が正だと波形は左にずれる



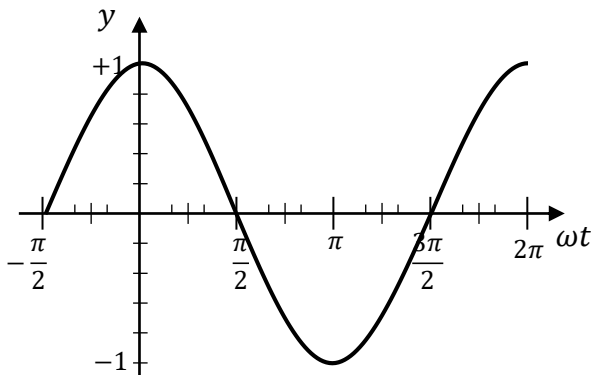
位相差が負だと波形は右にずれる



練習問題5

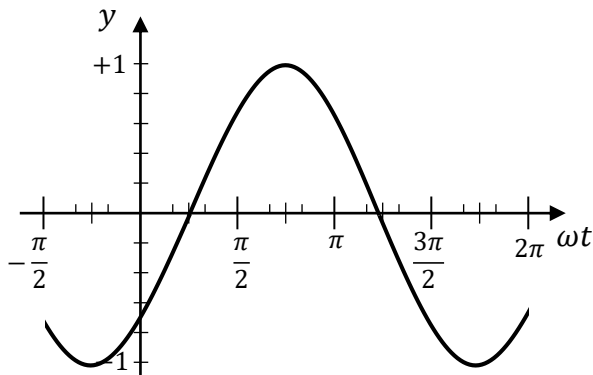
各問の位相差 ϕ の値を示せ。ただし、値は弧度法[rad]で表すこととする。

(1) $y = \sin(\omega t + \phi)$



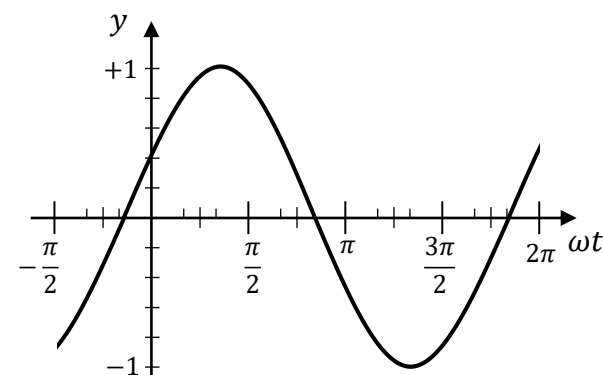
Ans. $\phi =$ _____

(2) $y = \sin(\omega t + \phi)$



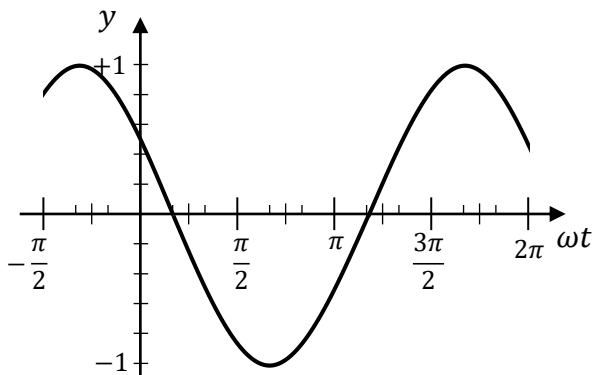
Ans. $\phi =$ _____

(3) $y = \sin(\omega t + \phi)$



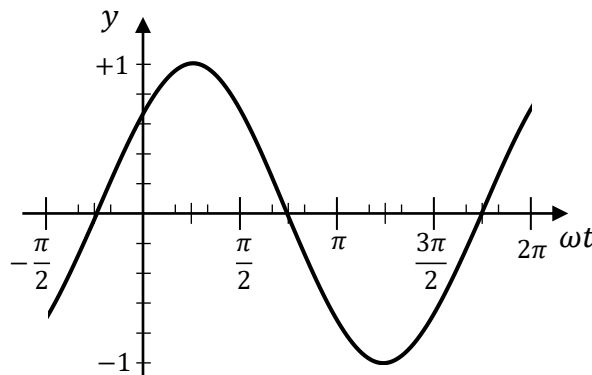
Ans. $\phi =$ _____

(4) $y = \cos(\omega t + \phi)$



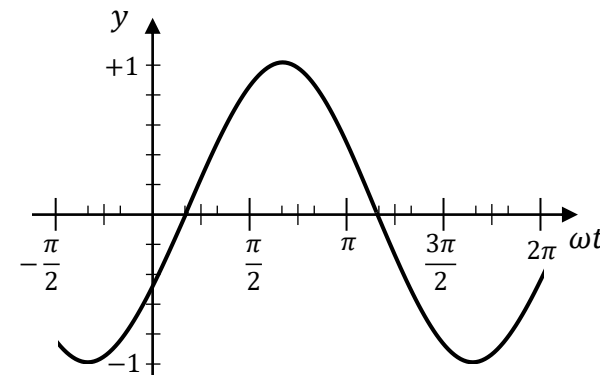
Ans. $\phi =$ _____

(5) $y = \cos(\omega t + \phi)$



Ans. $\phi =$ _____

(6) $y = \cos(\omega t + \phi)$

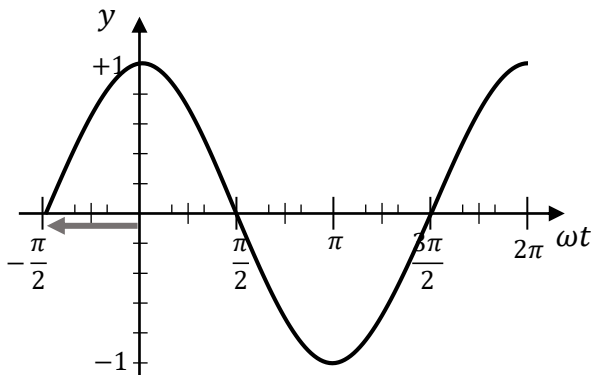


Ans. $\phi =$ _____

練習問題5 (解答)

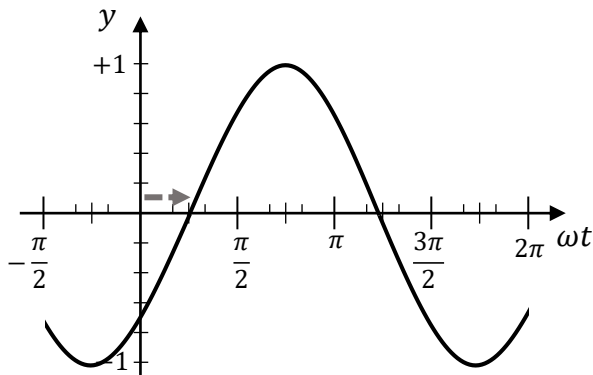
各問の位相差 ϕ の値を示せ。ただし、値は弧度法[rad]で表すこととする。

(1) $y = \sin(\omega t + \phi)$



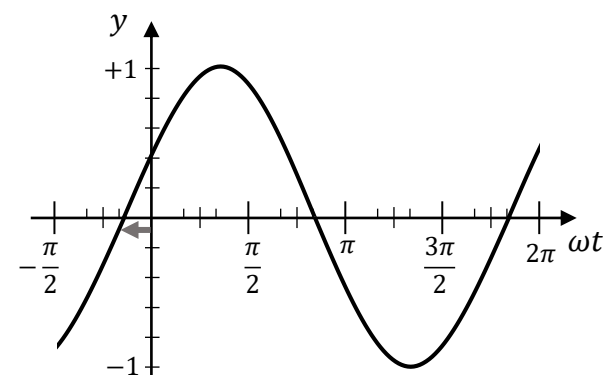
Ans. $\phi = \frac{\pi}{2}$

(2) $y = \sin(\omega t + \phi)$



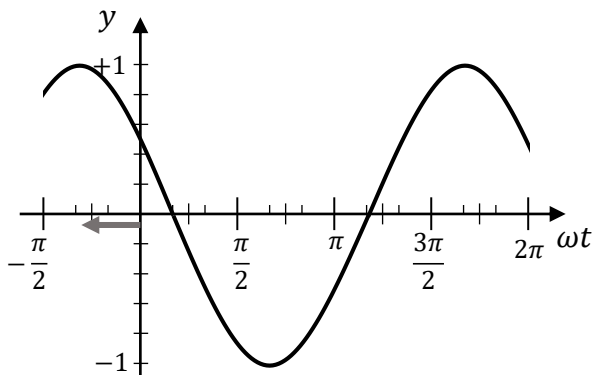
Ans. $\phi = -\frac{\pi}{4}$

(3) $y = \sin(\omega t + \phi)$



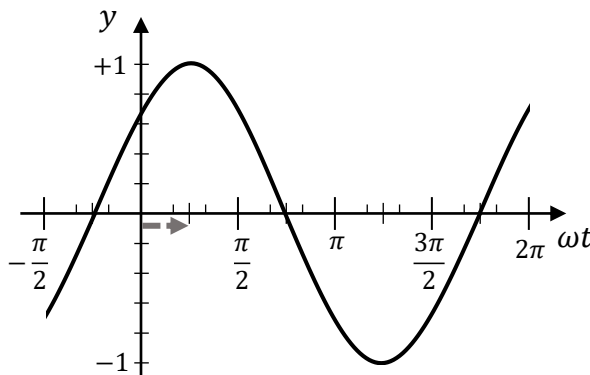
Ans. $\phi = \frac{\pi}{6}$

(4) $y = \cos(\omega t + \phi)$



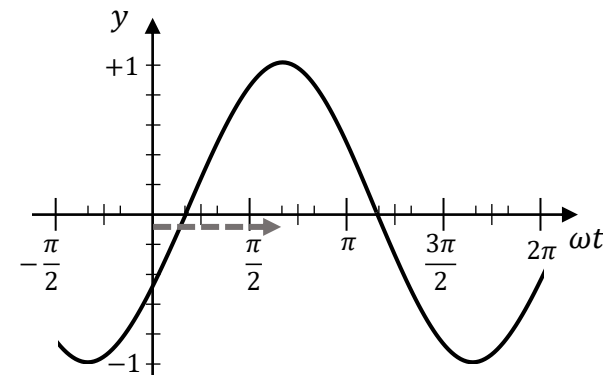
Ans. $\phi = \frac{\pi}{3}$

(5) $y = \cos(\omega t + \phi)$



Ans. $\phi = -\frac{\pi}{4}$

(6) $y = \cos(\omega t + \phi)$

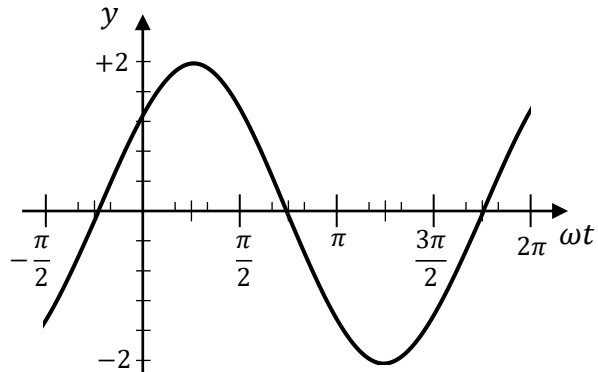


Ans. $\phi = -\frac{2\pi}{3}$

練習問題6

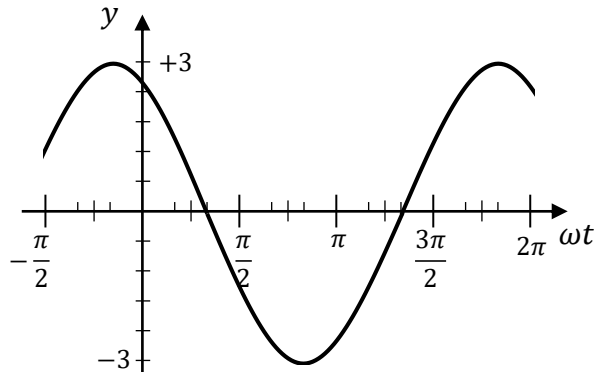
各問の振幅 A と位相差 ϕ の値を示せ。ただし、角度の値は弧度法[rad]で表すこととする。

(1) $y = A\sin(\omega t + \phi)$



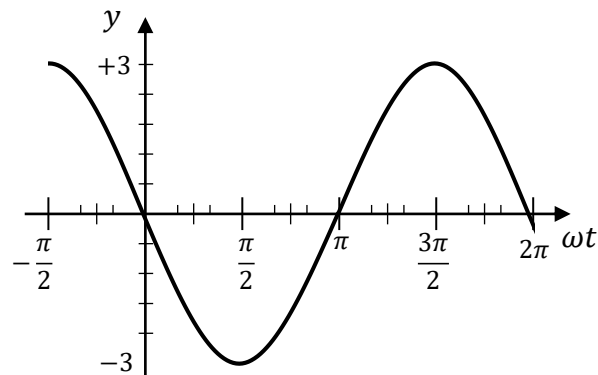
Ans. $A =$ $\phi =$

(2) $y = A\cos(\omega t + \phi)$



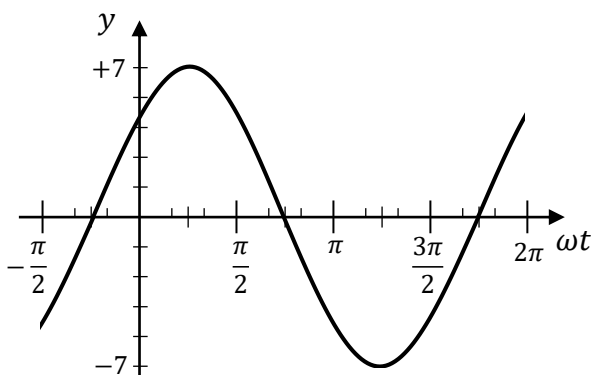
Ans. $A =$ $\phi =$

(3) $y = A\sin(\omega t + \phi)$



Ans. $A =$ $\phi =$

(4) $y = A\cos(\omega t + \phi)$

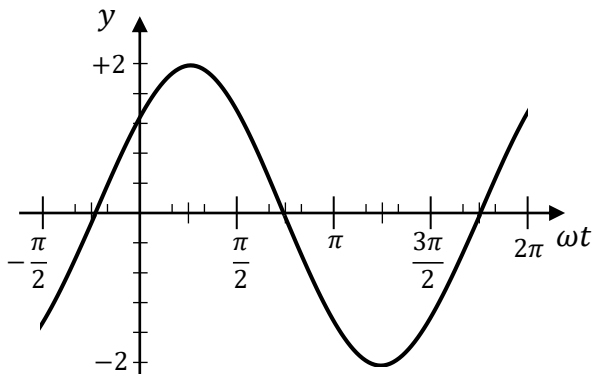


Ans. $A =$ $\phi =$

練習問題6 (解答)

各問の振幅 A と位相差 ϕ の値を示せ。ただし、角度の値は弧度法[rad]で表すこととする。

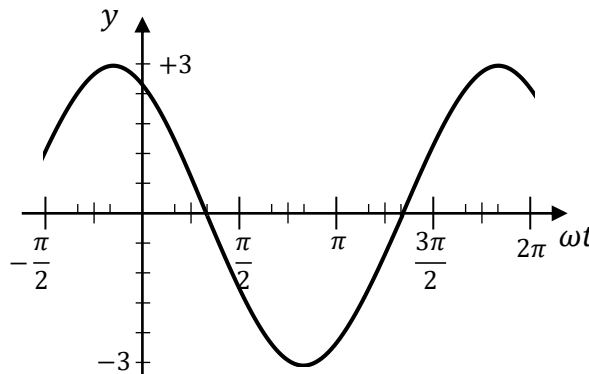
(1) $y = A\sin(\omega t + \phi)$



$$y = 2\sin\left(\omega t + \frac{\pi}{4}\right)$$

Ans. $A = 2$ $\phi = \frac{\pi}{4}$

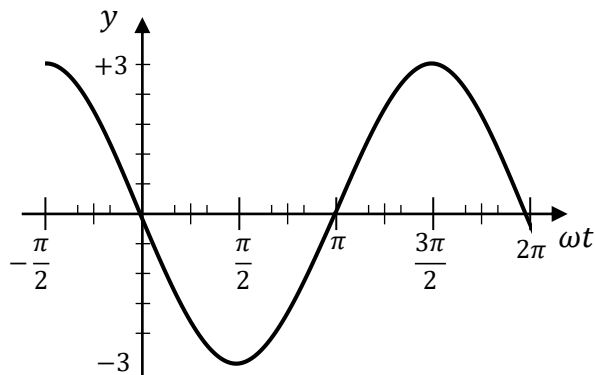
(2) $y = A\cos(\omega t + \phi)$



$$y = 3\cos\left(\omega t + \frac{\pi}{6}\right)$$

Ans. $A = 3$ $\phi = \frac{\pi}{6}$

(3) $y = A\sin(\omega t + \phi)$

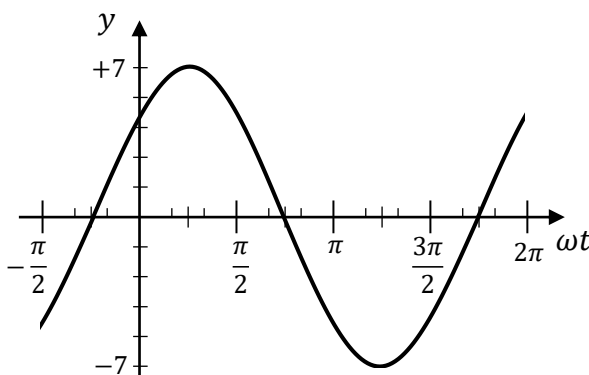


$$y = 3\sin(\omega t + \pi)$$

または、
 $y = 3\sin(\omega t - \pi)$
 $y = -3\sin \omega t$

Ans. $A = 3$ $\phi = \pi$

(4) $y = A\cos(\omega t + \phi)$



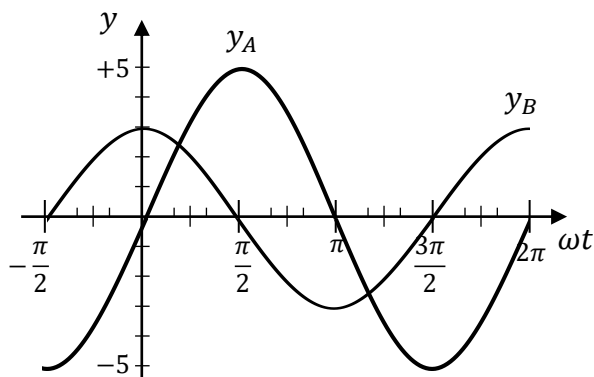
$$y = 7\cos\left(\omega t - \frac{\pi}{4}\right)$$

Ans. $A = 7$ $\phi = -\frac{\pi}{4}$

練習問題7

波形 y_A と y_B の振幅 A, B と位相差 $\phi_A, \phi_B, \phi_B - \phi_A$ の値を示せ。
ただし、角度の値は弧度法[rad]で表すこととする。

(1) $y_A = A\sin(\omega t + \phi_A), \quad y_B = B\sin(\omega t + \phi_B)$

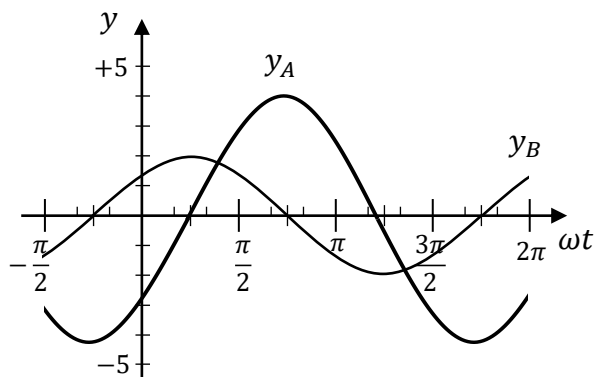


$y_A =$
 $y_B =$
 $\phi_B - \phi_A =$

$A = \quad B =$

Ans. $\phi_A = \quad \phi_B = \quad \phi_B - \phi_A =$

(2) $y_A = A\sin(\omega t + \phi_A), \quad y_B = B\sin(\omega t + \phi_B)$

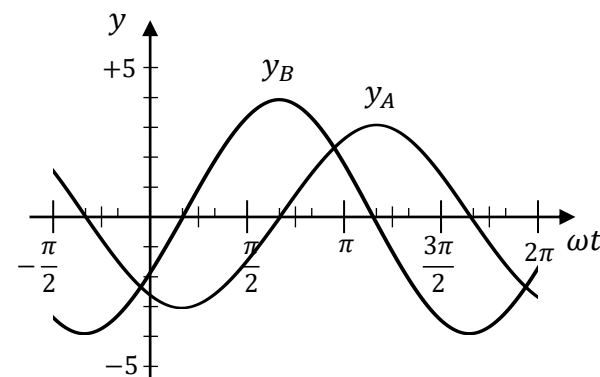


$y_A =$
 $y_B =$
 $\phi_B - \phi_A =$

$A = \quad B =$

Ans. $\phi_A = \quad \phi_B = \quad \phi_B - \phi_A =$

(3) $y_A = A\sin(\omega t + \phi_A), \quad y_B = B\sin(\omega t + \phi_B)$



$y_A =$
 $y_B =$
 $\phi_B - \phi_A =$

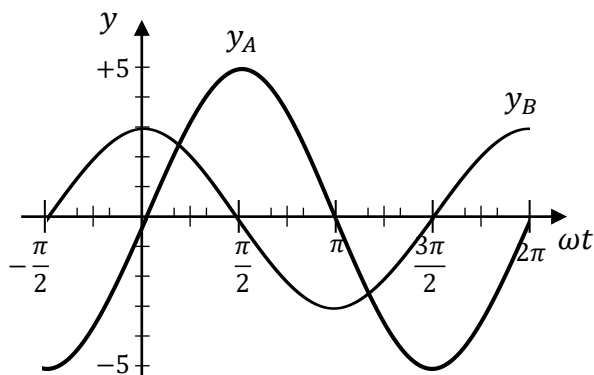
$A = \quad B =$

Ans. $\phi_A = \quad \phi_B = \quad \phi_B - \phi_A =$

練習問題7 (解答)

波形 y_A と y_B の振幅 A, B と位相差 $\phi_A, \phi_B, \phi_B - \phi_A$ の値を示せ。
ただし、角度の値は弧度法[rad]で表すこととする。

(1) $y_A = A\sin(\omega t + \phi_A), \quad y_B = B\sin(\omega t + \phi_B)$



$$y_A = 5\sin \omega t$$

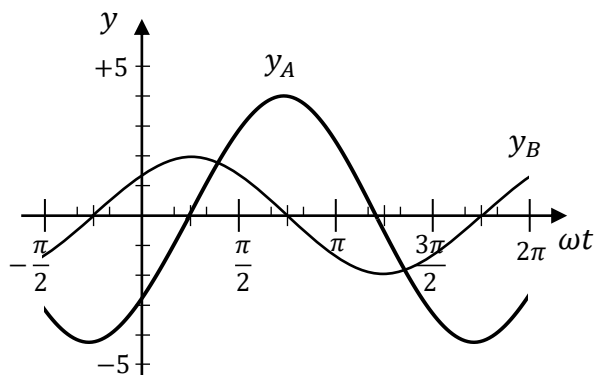
$$y_B = 3\sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\phi_B - \phi_A = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$A = 5 \quad B = 3$$

Ans. $\phi_A = 0 \quad \phi_B = \frac{\pi}{2} \quad \phi_B - \phi_A = \frac{\pi}{2}$

(2) $y_A = A\sin(\omega t + \phi_A), \quad y_B = B\sin(\omega t + \phi_B)$



$$y_A = 4\sin\left(\omega t - \frac{\pi}{4}\right)$$

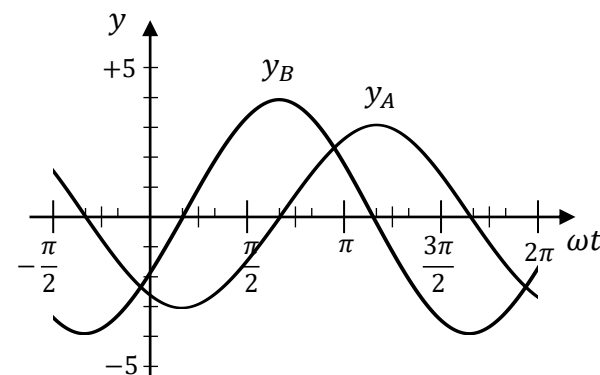
$$y_B = 2\sin\left(\omega t + \frac{\pi}{4}\right)$$

$$\phi_B - \phi_A = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

$$A = 4 \quad B = 2$$

Ans. $\phi_A = -\frac{\pi}{4} \quad \phi_B = \frac{\pi}{4} \quad \phi_B - \phi_A = \frac{\pi}{2}$

(3) $y_A = A\sin(\omega t + \phi_A), \quad y_B = B\sin(\omega t + \phi_B)$



$$y_A = 3\sin\left(\omega t - \frac{2\pi}{3}\right)$$

$$y_B = 4\sin\left(\omega t - \frac{\pi}{6}\right)$$

$$\phi_B - \phi_A = -\frac{\pi}{6} - \left(-\frac{2\pi}{3}\right) = -\frac{\pi}{6} + \frac{4\pi}{6} = \frac{\pi}{2}$$

$$A = 3 \quad B = 4$$

Ans. $\phi_A = -\frac{2\pi}{3} \quad \phi_B = -\frac{\pi}{6} \quad \phi_B - \phi_A = \frac{\pi}{2}$

ご聴講ありがとうございました
ございました!!