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KWG presents

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第12回 電気数学
複素数(2)

2022.11.26 Sat

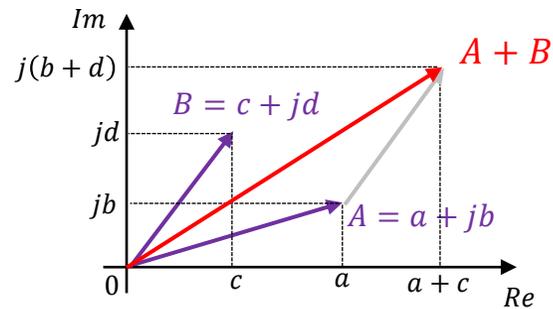
複素平面

複素平面：横軸を実数（実軸 Re ）、縦軸を虚数（虚軸 Im ）とした平面上で複素数を表現する

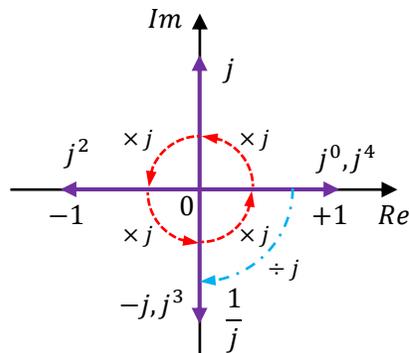
○複素平面における複素数の表現

複素平面で複素数を表現するとき、2次元平面上ベクトルと同じように考えてよい。

2つの複素数 $A = a + jb$, $B = c + jd$ は複素平面上で以下のように表すことができる。



2つの複素数の和 $A + B = a + c + j(b + d)$ は
2つのベクトルのベクトル和と同じように考えることができる。



虚数 j はべき乗することで複素平面上を回転する。

$$j^0 = (\sqrt{-1})^0 = 1$$

$$j^{-1} = \frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$$

$$j^2 = (\sqrt{-1})^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$$

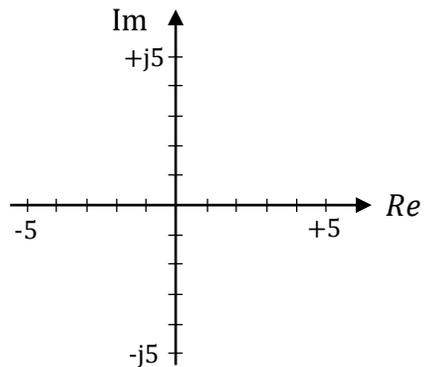
$$j^3 = (\sqrt{-1})^3 = (\sqrt{-1})^2 \cdot \sqrt{-1} = (-1) \cdot j = -j$$

$$j^4 = (\sqrt{-1})^4 = (\sqrt{-1})^2 \cdot (\sqrt{-1})^2 = (-1) \cdot (-1) = 1$$

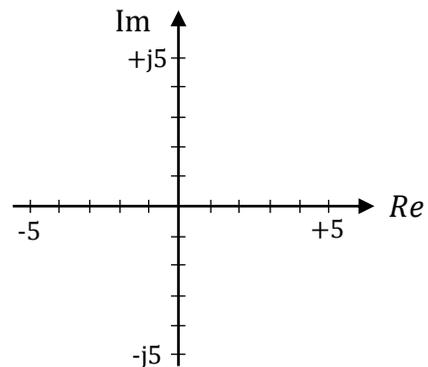
練習問題 I

各問で与えられる複素数 z を複素平面上に示せ。

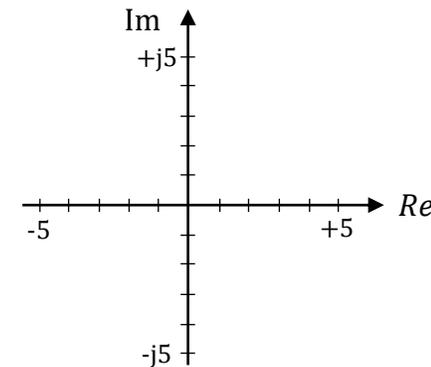
(1) $z = 2 + j2$



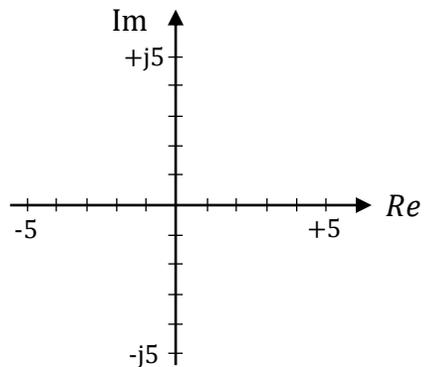
(2) $z = 1 + j3$



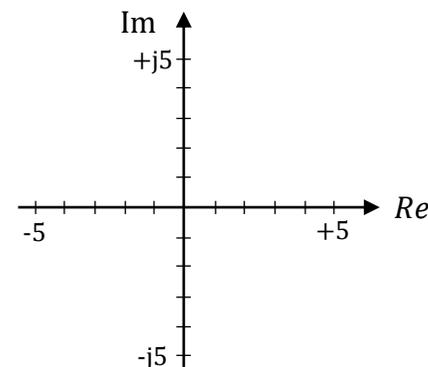
(3) $z = (2 + j2) + (1 + j3)$



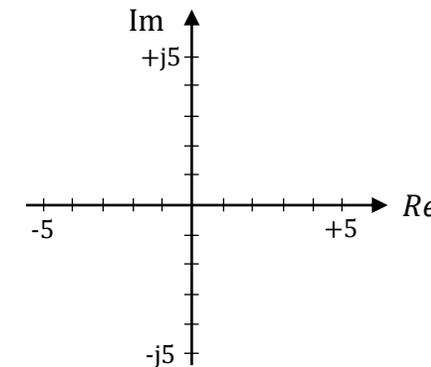
(4) $z = 3 + j$



(5) $z = -2 - j4$



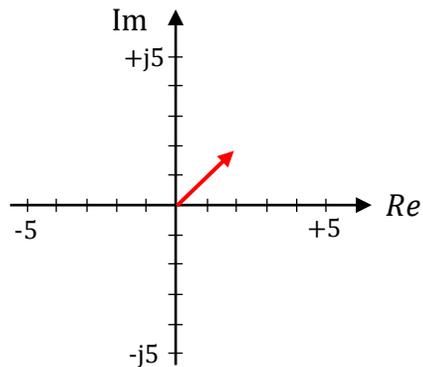
(6) $z = (3 + j) - (2 + j4)$



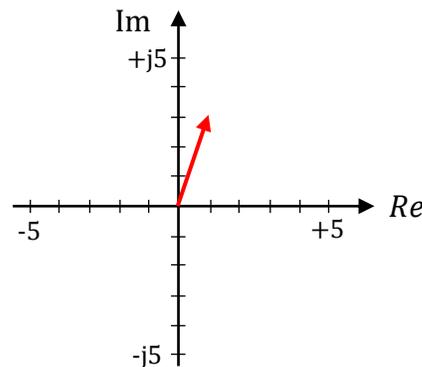
練習問題 I (解答)

各問で与えられる複素数 z を複素平面上に示せ。

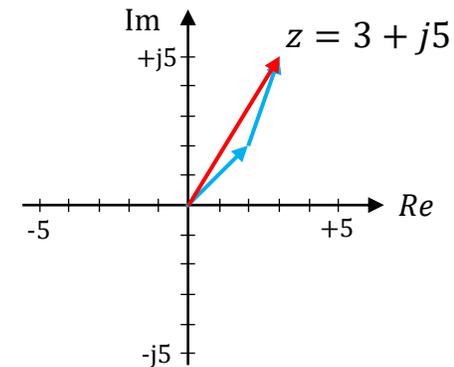
(1) $z = 2 + j2$



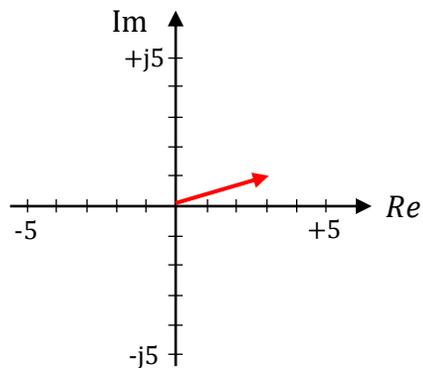
(2) $z = 1 + j3$



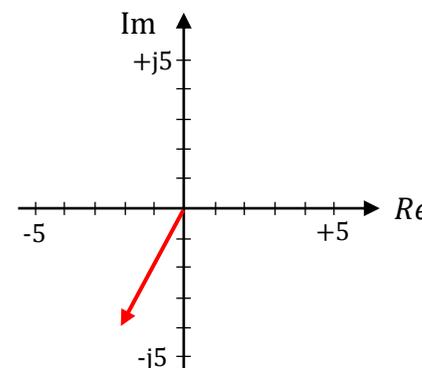
(3) $z = (2 + j2) + (1 + j3)$



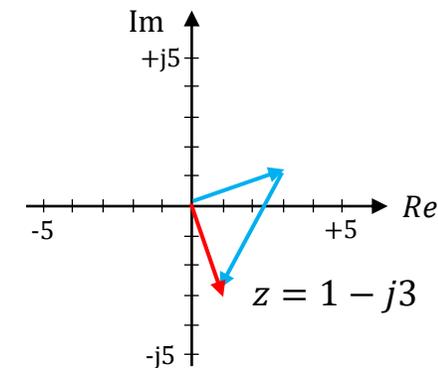
(4) $z = 3 + j$



(5) $z = -2 - j4$



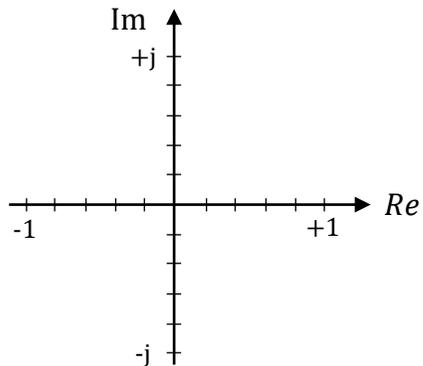
(6) $z = (3 + j) - (2 + j4)$



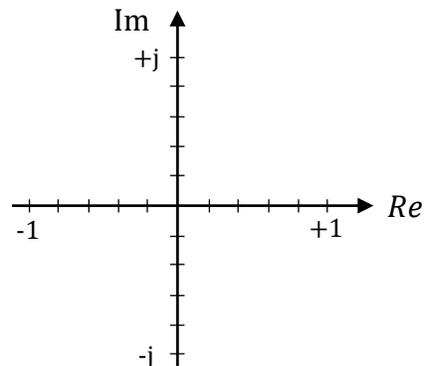
練習問題2

各問で与えられる複素数 z を複素平面上に示せ。

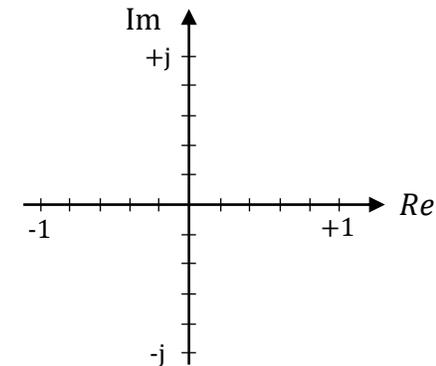
(1) $z = j$



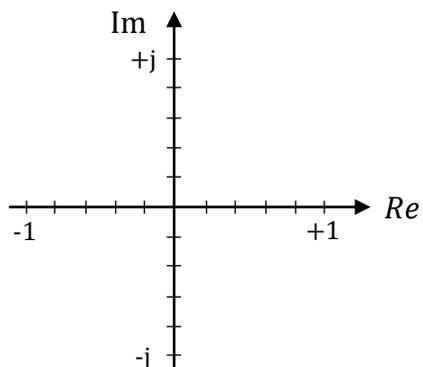
(2) $z = j^2$



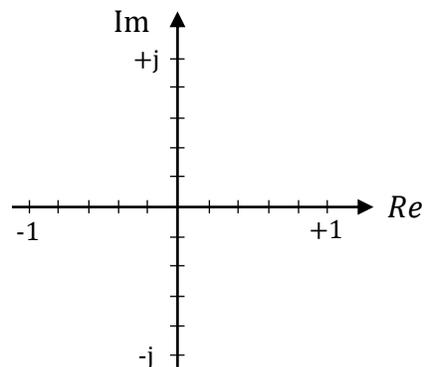
(3) $z = j^3$



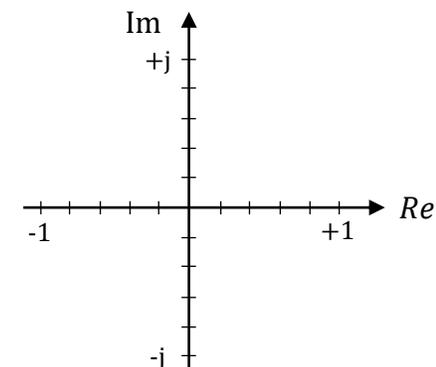
(4) $z = j^4$



(5) $z = \frac{1}{j}$



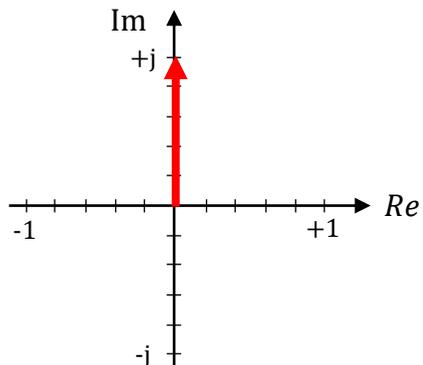
(6) $z = j^{-4}$



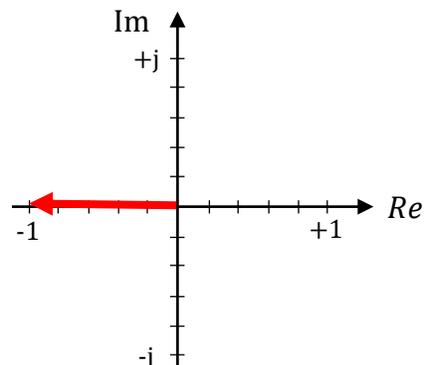
練習問題2 (解答)

各問で与えられる複素数 z を複素平面上に示せ。

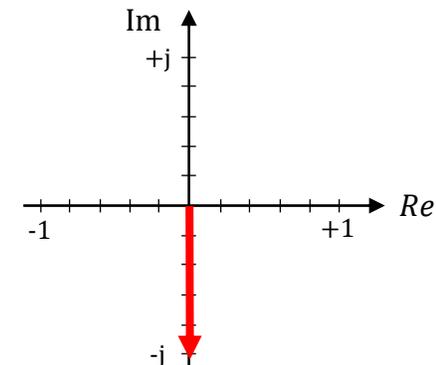
(1) $z = j$



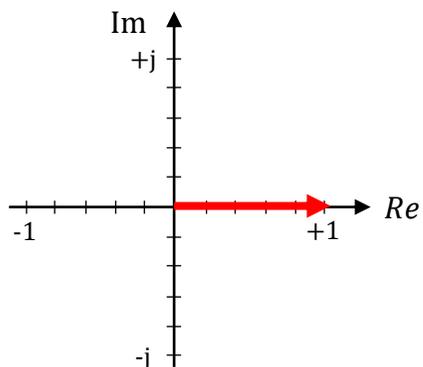
(2) $z = j^2 = -1$



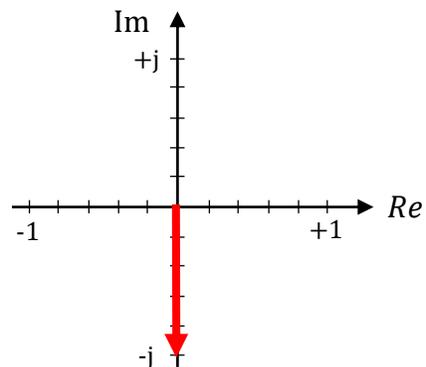
(3) $z = j^3 = -1 \times j = -j$



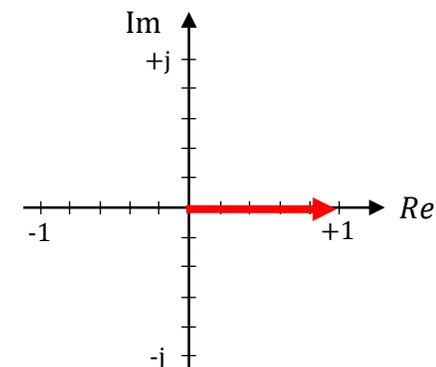
(4) $z = j^4 = -1 \times (-1) = 1$



(5) $z = \frac{1}{j} = \frac{j}{j \times j} = \frac{j}{-1} = -j$



(6) $z = j^{-4} = \frac{1}{j^4} = \frac{1}{1} = 1$



複素数の表現 (複素数表示とフェーザ表示)

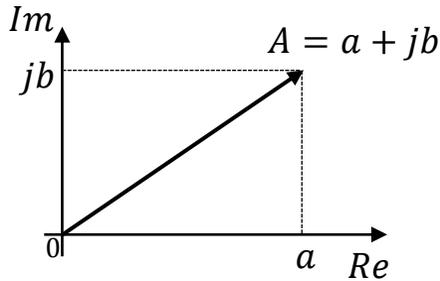
複素数 $A = a + jb$ という表現を複素数表示というのに対し、複素数の絶対値と実軸を基準にした角度で複素数を表現することをフェーザ表示という。

$$A = a + jb \leftrightarrow A = r \angle \theta$$

複素数表示 フェーザ表示

複素数表示とフェーザ表示は同じ意味を持ち、互いの表現に変換することが可能
 <複素数表示からフェーザ表示>

$A = a + jb$ をフェーザ表示に変換する



複素数表示

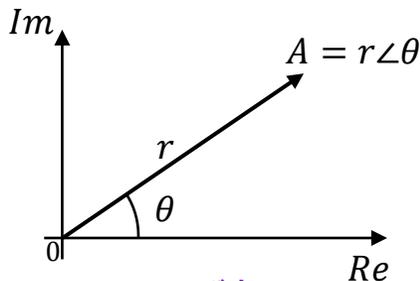
$$r = |A| = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1} \frac{b}{a} \rightarrow \frac{b}{a} = \tan \theta$$

$$A = r \angle \theta = \sqrt{a^2 + b^2} \angle \tan^{-1} \frac{b}{a}$$

<フェーザ表示から複素数表示>

$A = r \angle \theta$ を複素数表示に変換する



フェーザ表示

$$a = r \cos \theta$$

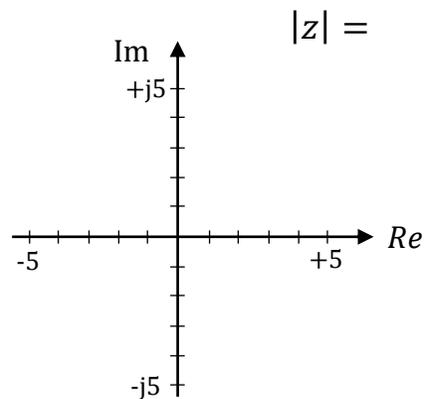
$$b = r \sin \theta$$

$$A = a + jb = r(\cos \theta + j \sin \theta)$$

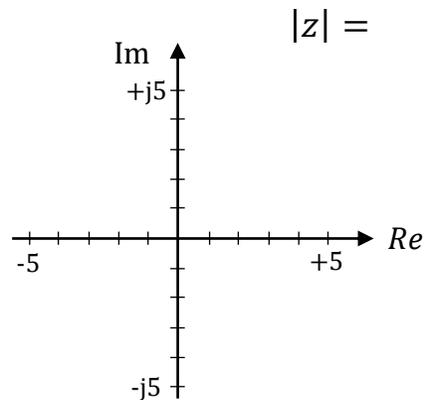
練習問題3

各問で与えられる複素数 z を複素平面上に示せ。また、絶対値を $|z|$ 導出せよ。

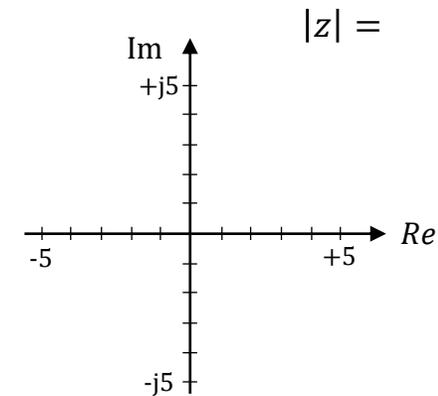
(1) $z = 3 + j4$



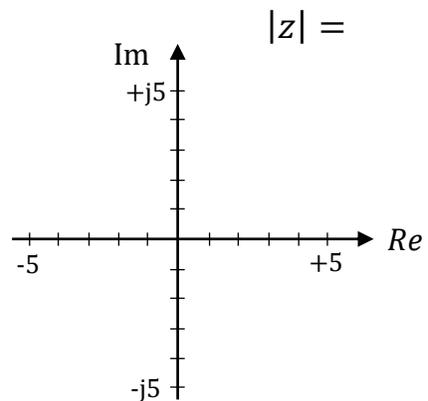
(2) $z = 2 + j2$



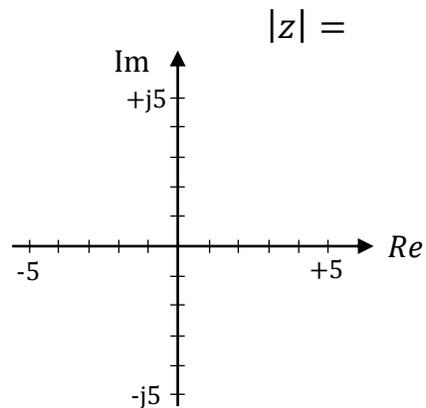
(3) $z = 1 + j\sqrt{3}$



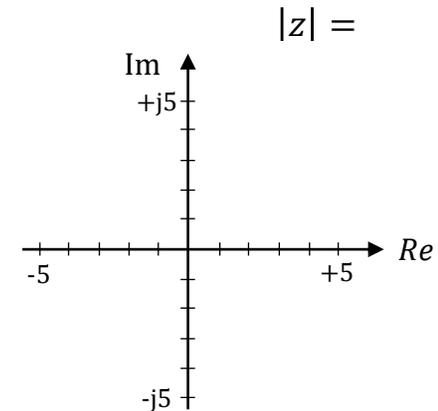
(4) $z = -2 + j4$



(5) $z = -2 - j4$



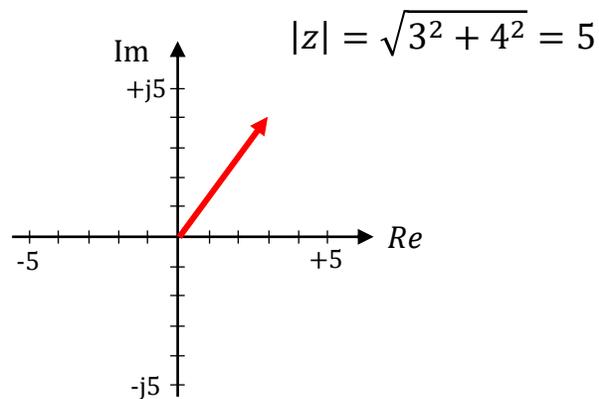
(6) $z = (5 + j5) - (2 + j4)$



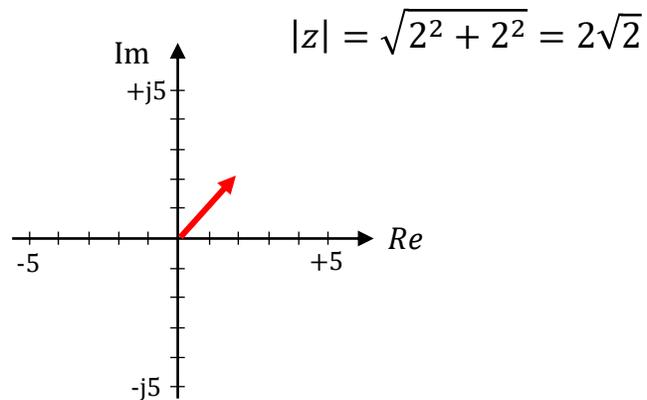
練習問題3 (解答)

各問で与えられる複素数 z を複素平面上に示せ。また、絶対値を $|z|$ 導出せよ。

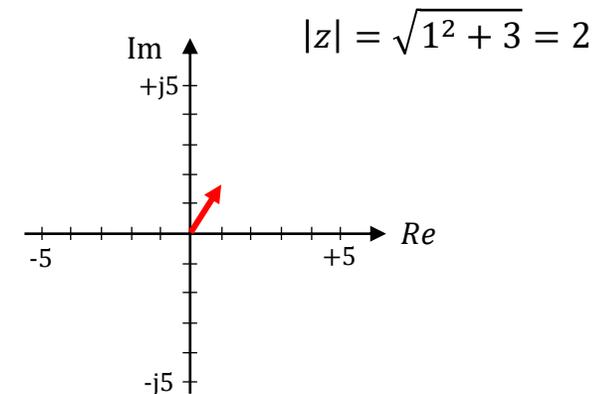
(1) $z = 3 + j4$



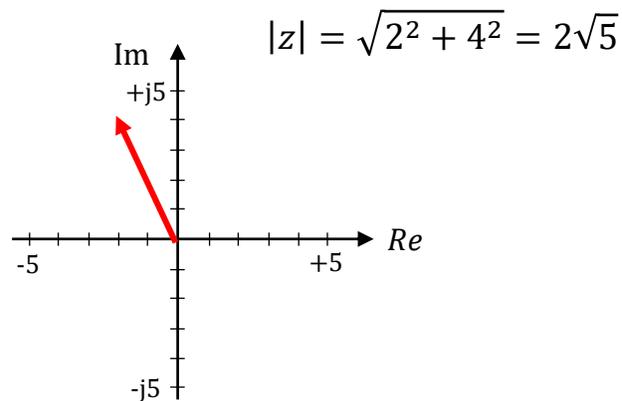
(2) $z = 2 + j2$



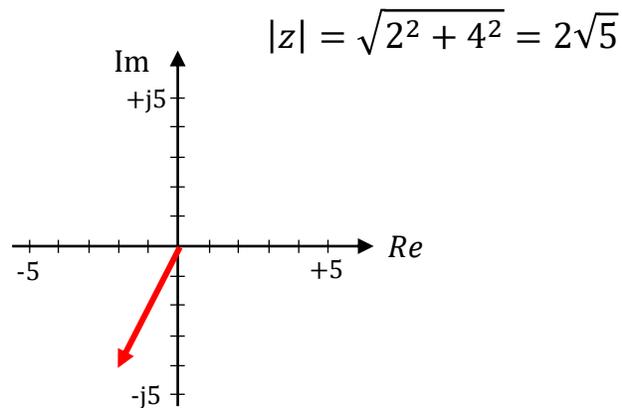
(3) $z = 1 + j\sqrt{3}$



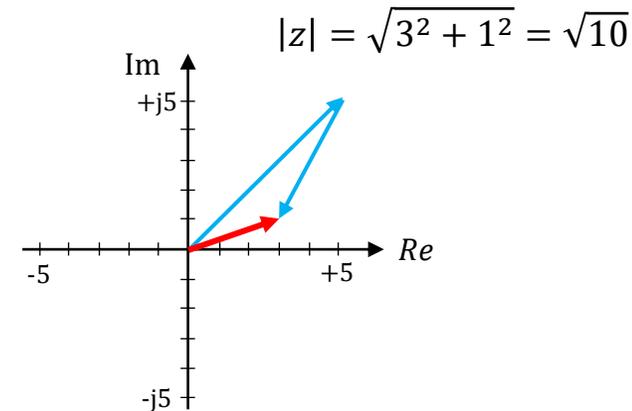
(4) $z = -2 + j4$



(5) $z = -2 - j4$



(6) $z = (5 + j5) - (2 + j4)$



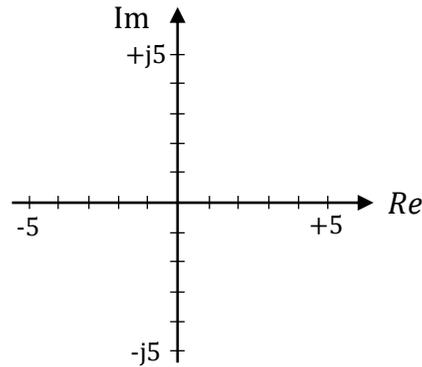
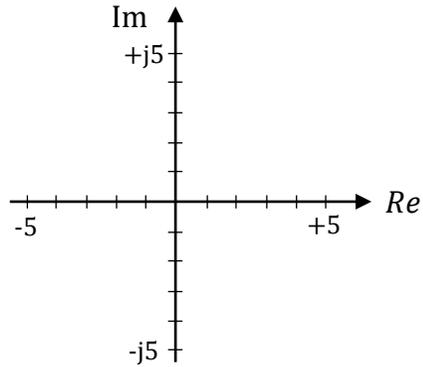
練習問題4

各問で与えられる複素数 z を複素平面上に示せ。また、偏角を導出せよ。

(1) $z = 2 + j2$

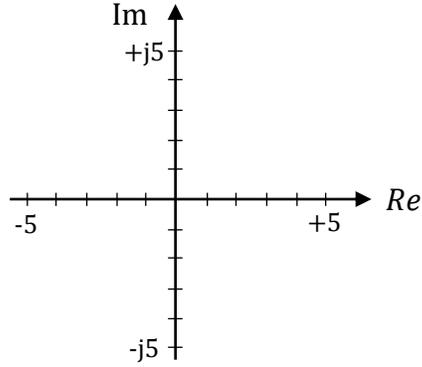
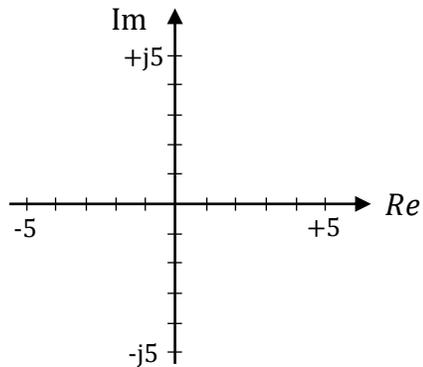
(2) $z = 1.5 + j4$

(偏角は表を参照し最も近い値を選ぶこと)



(3) $z = 5 + j3$

(4) $z = 2.5 - j3$



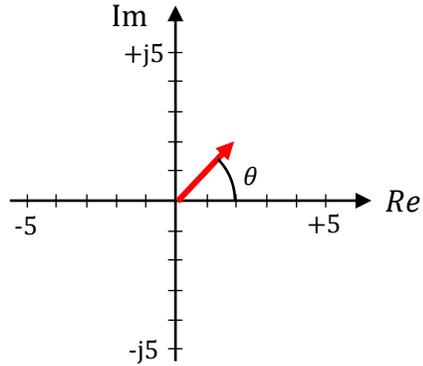
$\theta (^{\circ})$	$\tan \theta$
0	0.00
10	0.18
20	0.36
30	0.58
40	0.84
45	1.00
50	1.19
60	1.73
70	2.75
80	5.67
90	—

練習問題4 (解答)

各問で与えられる複素数 z を複素平面上に示せ。また、偏角を導出せよ。

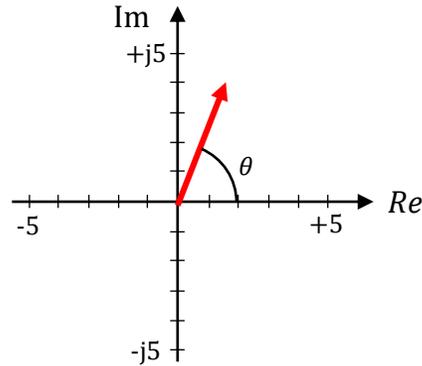
(1) $z = 2 + j2$

$$\tan \theta = \frac{2}{2} = 1 \rightarrow \theta = 45^\circ$$



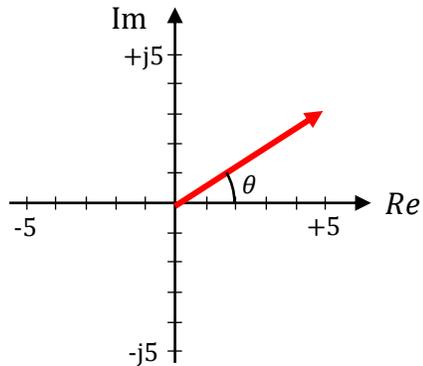
(2) $z = 1.5 + j4$

$$\tan \theta = \frac{4}{1.5} = 2.67 \rightarrow \theta \sim 70^\circ$$



(3) $z = 5 + j3$

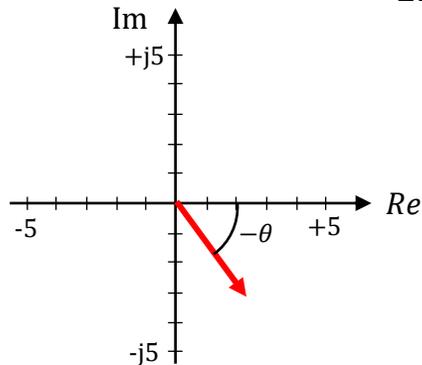
$$\tan \theta = \frac{3}{5} = 0.6 \rightarrow \theta \sim 30^\circ$$



(4) $z = 2.5 - j3$

$$\tan \theta = \frac{-3}{2.5} = -1.2 \rightarrow -\theta \sim 50^\circ$$

$$\theta \sim -50^\circ$$

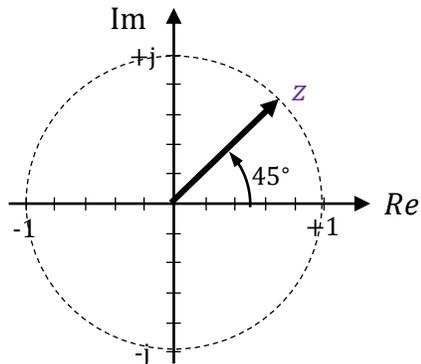


$\theta (^{\circ})$	$\tan \theta$
0	0.00
10	0.18
20	0.36
30	0.58
40	0.84
45	1.00
50	1.19
60	1.73
70	2.75
80	5.67
90	—

練習問題5

複素数表示と指数関数表示で以下の z を示せ。 z の大きさは1とする。

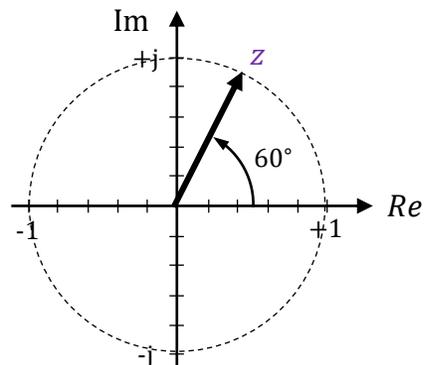
(1)



Ans. 複素数表示：

フェーザ表示：

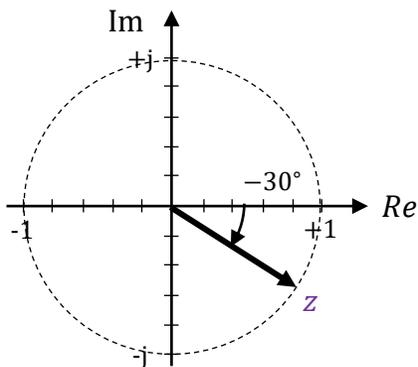
(2)



Ans. 複素数表示：

フェーザ表示：

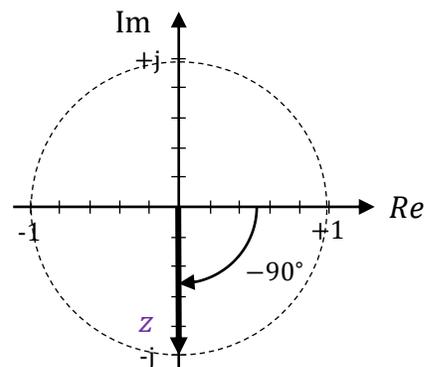
(3)



Ans. 複素数表示：

フェーザ表示：

(4)



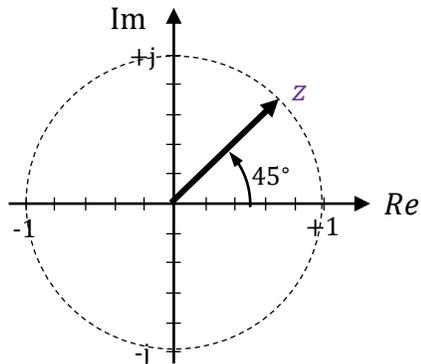
Ans. 複素数表示：

フェーザ表示：

練習問題5 (解答)

複素数表示と指数関数表示で以下の z を示せ。 z の大きさは1とする。

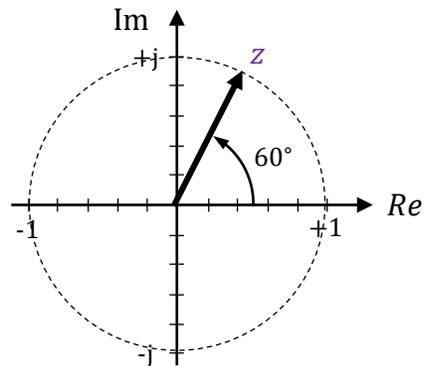
(1)



$$\begin{aligned} z &= \cos 45^\circ + j \sin 45^\circ \\ &= \frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \\ z &= 1 \angle 45^\circ \end{aligned}$$

Ans. 複素数表示: $z = \frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}}$ フェーザ表示: $z = 1 \angle 45^\circ$

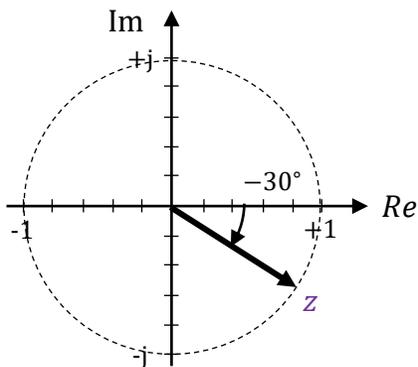
(2)



$$\begin{aligned} z &= \cos 60^\circ + j \sin 60^\circ \\ &= \frac{1}{2} + j \frac{\sqrt{3}}{2} \\ z &= 1 \angle 60^\circ \end{aligned}$$

Ans. 複素数表示: $z = \frac{1}{2} + j \frac{\sqrt{3}}{2}$ フェーザ表示: $z = 1 \angle 60^\circ$

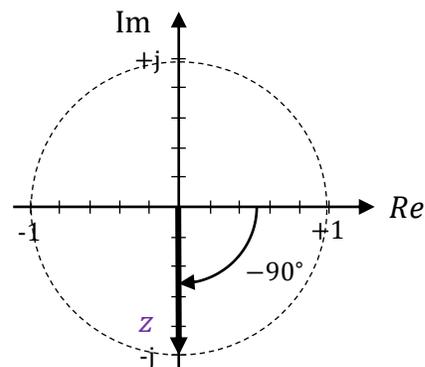
(3)



$$\begin{aligned} z &= \cos(-30^\circ) + j \sin(-30^\circ) \\ &= \frac{\sqrt{3}}{2} - j \frac{1}{2} \\ z &= 1 \angle -30^\circ \end{aligned}$$

Ans. 複素数表示: $z = \frac{\sqrt{3}}{2} - j \frac{1}{2}$ フェーザ表示: $z = 1 \angle -30^\circ$

(4)



$$\begin{aligned} z &= \cos(-90^\circ) + j \sin(-90^\circ) \\ &= 0 - j \\ z &= 1 \angle -90^\circ \end{aligned}$$

Ans. 複素数表示: $z = -j$ フェーザ表示: $z = 1 \angle -90^\circ$

ご聴講ありがとうございました
ございました!!