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KWG presents

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第9回 電気数学
三角関数(1)

2022.10.29 Sat

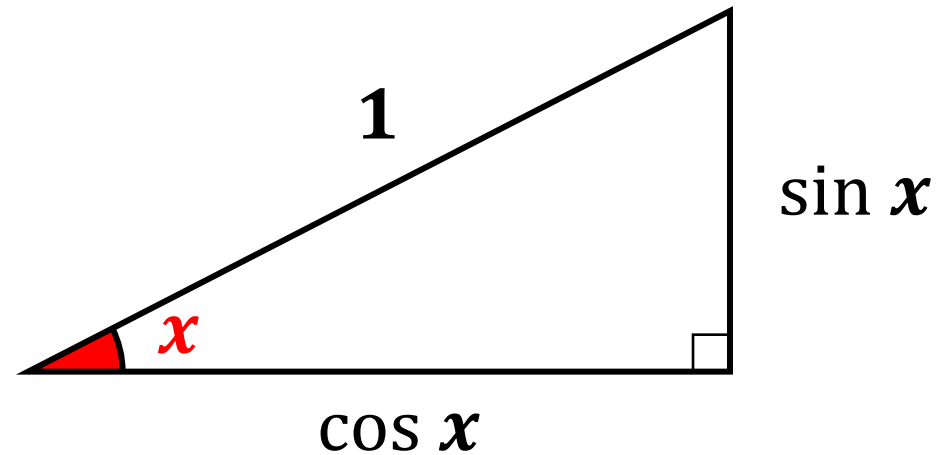
三角関数

直角三角形の1つの角を変数 x で表し、 x に対する三角比の値を y とした関数を“三角関数”という

$$y = \sin x$$

$$y = \cos x$$

$$y = \tan x$$



<三角関数に慣れるために必要な数学の土台>

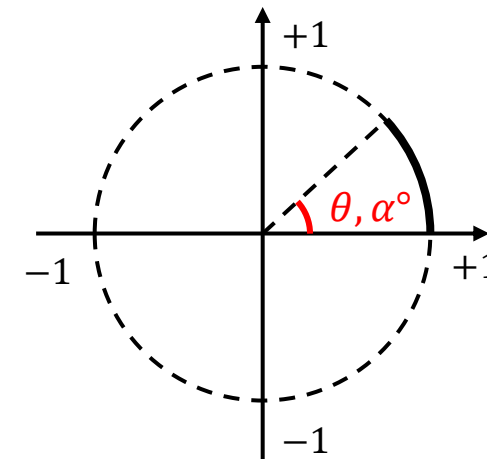
- 度数法と弧度法による角度の表現
- 三平方の定理
- 三角比

度数法と弧度法

○度数法と弧度法

角の大きさを表す際に、度 [°] を用いる方法を度数法という。

半径1の単位円の円周の長さで角を表す方法を弧度法といい、この角の大きさはラジアン [rad] という単位を用いる。円の1周の角360°を弧度法で表すと2π [rad] となる。



$$\theta \text{ [rad]} = \alpha^\circ \times \frac{2\pi}{360^\circ}$$

度 [°]	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
ラジアン [rad]	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π

練習問題 I

次の角度を弧度法で表せ。

(1) 30°

(2) 45°

(3) 90°

(4) 135°

Ans. _____

Ans. _____

Ans. _____

Ans. _____

(5) 150°

(6) 225°

(7) 315°

(8) 330°

Ans. _____

Ans. _____

Ans. _____

Ans. _____

練習問題Ⅰ (解答)

次の角度を弧度法で表せ。

(1) 30°

$$\frac{30^\circ}{180^\circ} \times \pi = \frac{1}{6}\pi$$

Ans. $\frac{1}{6}\pi$ [rad]

(2) 45°

$$\frac{45^\circ}{180^\circ} \times \pi = \frac{1}{4}\pi$$

Ans. $\frac{1}{4}\pi$ [rad]

(3) 90°

$$\frac{90^\circ}{180^\circ} \times \pi = \frac{1}{2}\pi$$

Ans. $\frac{1}{2}\pi$ [rad]

(4) 135°

$$\frac{135^\circ}{180^\circ} \times \pi = \frac{3}{4}\pi$$

Ans. $\frac{3}{4}\pi$ [rad]

(5) 150°

$$\frac{150^\circ}{180^\circ} \times \pi = \frac{5}{6}\pi$$

Ans. $\frac{5}{6}\pi$ [rad]

(6) 225°

$$\frac{225^\circ}{180^\circ} \times \pi = \frac{5}{4}\pi$$

Ans. $\frac{5}{4}\pi$ [rad]

(7) 315°

$$\frac{315^\circ}{180^\circ} \times \pi = \frac{7}{4}\pi$$

Ans. $\frac{7}{4}\pi$ [rad]

(8) 330°

$$\frac{330^\circ}{180^\circ} \times \pi = \frac{11}{6}\pi$$

Ans. $\frac{11}{6}\pi$ [rad]

練習問題2

次の角度を度数法で表せ。

(1) $\frac{1}{2}\pi$

Ans. _____

(2) $\frac{3}{4}\pi$

Ans. _____

(3) $\frac{1}{3}\pi$

Ans. _____

(4) $\frac{7}{6}\pi$

Ans. _____

(5) $\frac{5}{3}\pi$

Ans. _____

(6) $\frac{11}{6}\pi$

Ans. _____

(7) $\frac{20}{3}\pi$

Ans. _____

(8) $\frac{17}{6}\pi$

Ans. _____

練習問題2 (解答)

次の角度を度数法で表せ。

(1) $\frac{1}{2}\pi$

$$\frac{1}{2}\pi \times \frac{180^\circ}{\pi} = 90^\circ$$

Ans. 90°

(2) $\frac{3}{4}\pi$

$$\frac{3}{4}\pi \times \frac{180^\circ}{\pi} = 135^\circ$$

Ans. 135°

(3) $\frac{1}{3}\pi$

$$\frac{1}{3}\pi \times \frac{180^\circ}{\pi} = 60^\circ$$

Ans. 60°

(4) $\frac{7}{6}\pi$

$$\frac{7}{6}\pi \times \frac{180^\circ}{\pi} = 210^\circ$$

Ans. 210°

(5) $\frac{5}{3}\pi$

$$\frac{5}{3}\pi \times \frac{180^\circ}{\pi} = 300^\circ$$

Ans. 300°

(6) $\frac{11}{6}\pi$

$$\frac{11}{6}\pi \times \frac{180^\circ}{\pi} = 330^\circ$$

Ans. 330°

(7) $\frac{20}{3}\pi$

$$\frac{20}{3}\pi \times \frac{180^\circ}{\pi} = 1200^\circ$$

Ans. 1200°

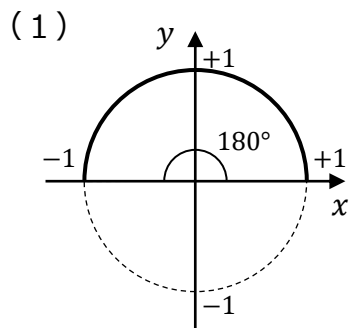
(8) $\frac{17}{6}\pi$

$$\frac{17}{6}\pi \times \frac{180^\circ}{\pi} = 510^\circ$$

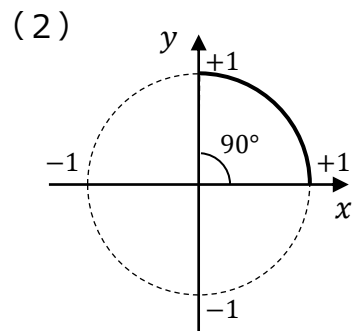
Ans. 510°

練習問題3

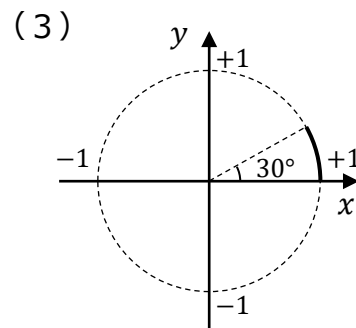
各問の実線で示す部分の円周の長さを求めよ。



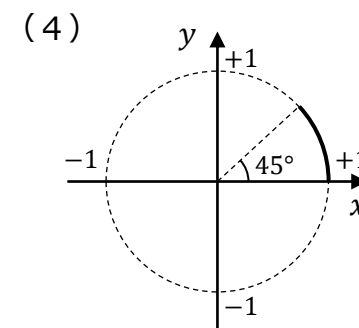
Ans. _____



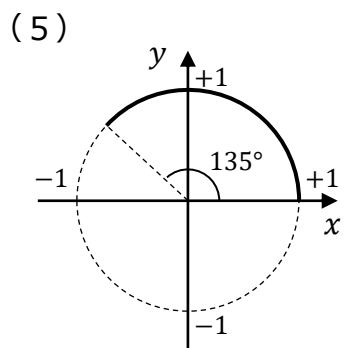
Ans. _____



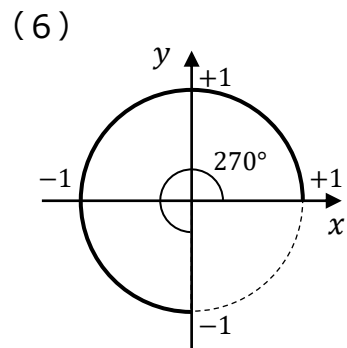
Ans. _____



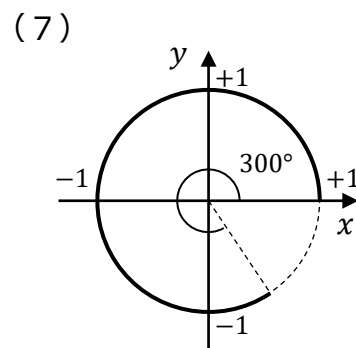
Ans. _____



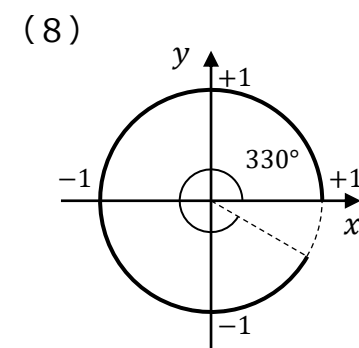
Ans. _____



Ans. _____



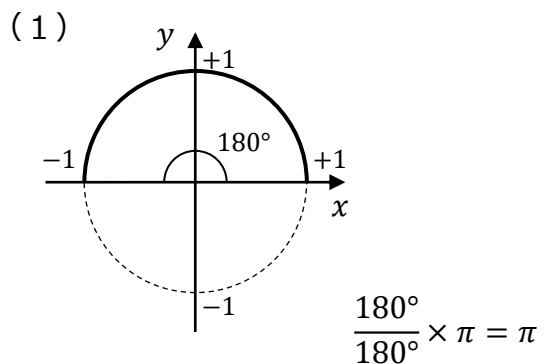
Ans. _____



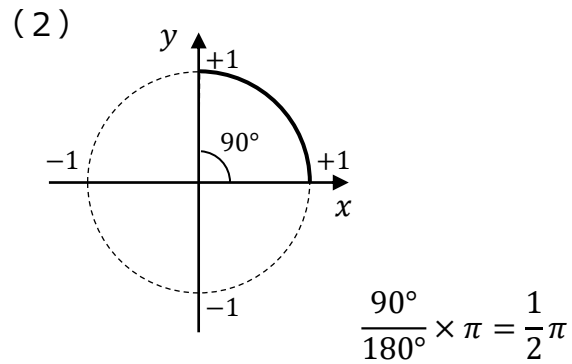
Ans. _____

練習問題3 (解答)

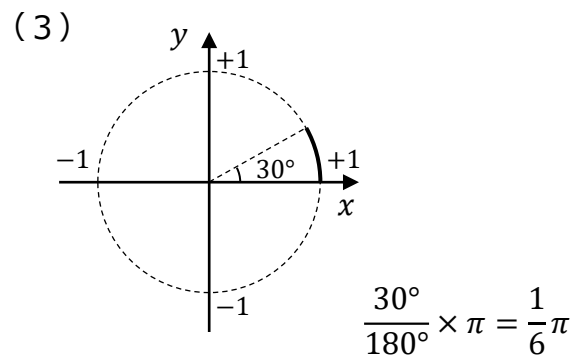
各問の実線で示す部分の円周の長さを求めよ。



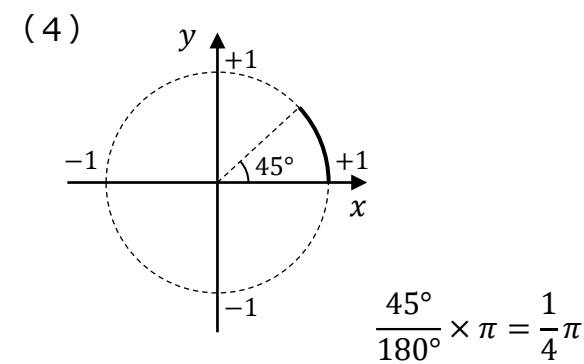
Ans. π



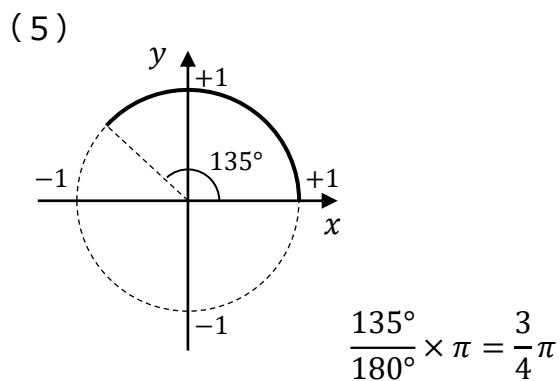
Ans. $\frac{1}{2}\pi$



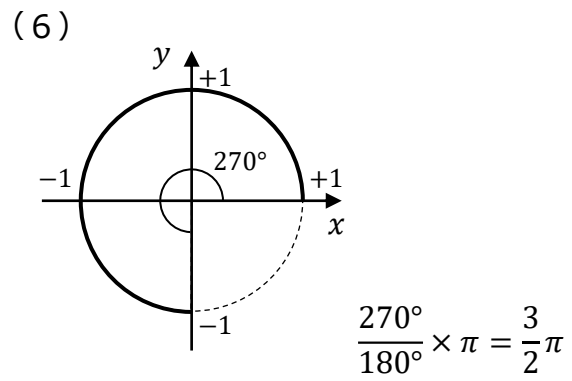
Ans. $\frac{1}{6}\pi$



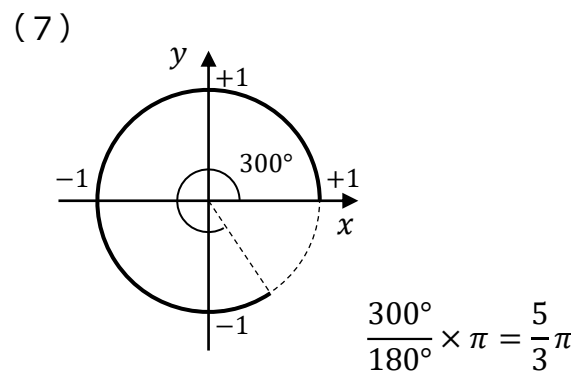
Ans. $\frac{1}{4}\pi$



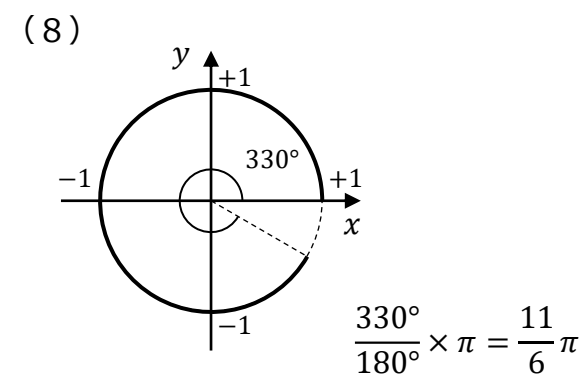
Ans. $\frac{3}{4}\pi$



Ans. $\frac{3}{2}\pi$

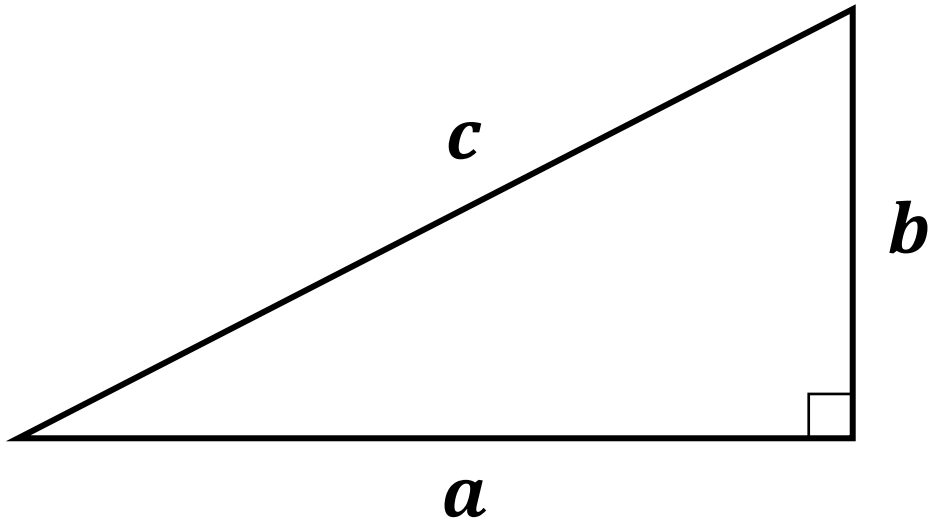


Ans. $\frac{5}{3}\pi$



Ans. $\frac{11}{6}\pi$

三平方の定理



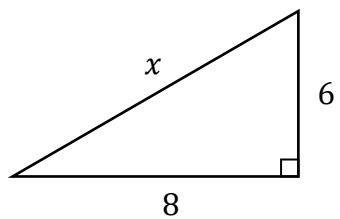
2辺の長さをa, b, 斜辺の長さをcとする
直角三角形において次式が成り立つ。

$$c^2 = a^2 + b^2$$
$$c = \sqrt{a^2 + b^2}$$

- 直角三角形に対する公式
- 直角三角形の各辺の長さの関係を表す
- 直角を作る2つの辺の長さとして直角と向かい合う辺 (斜辺) の関係を表す

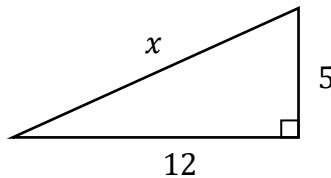
練習問題4

(1)



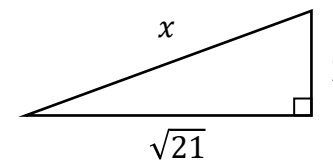
Ans. $x =$ _____

(2)



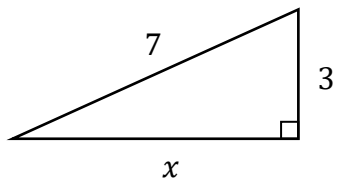
Ans. $x =$ _____

(3)



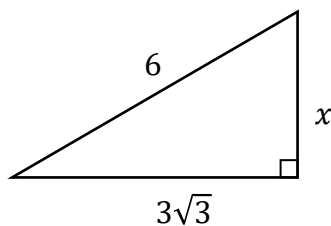
Ans. $x =$ _____

(4)



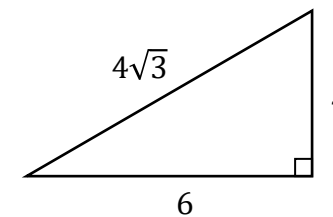
Ans. $x =$ _____

(5)



Ans. $x =$ _____

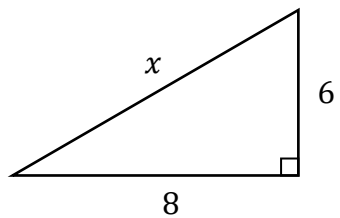
(6)



Ans. $x =$ _____

練習問題4 (解答)

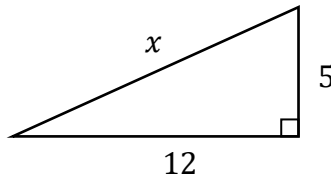
(1)



$$\begin{aligned}x^2 &= 6^2 + 8^2 = 100 \\x^2 &= 10^2 \\x &= 10\end{aligned}$$

Ans. $x = 10$

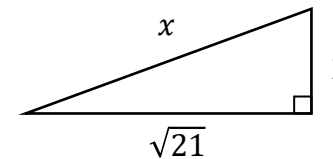
(2)



$$\begin{aligned}x^2 &= 5^2 + 12^2 = 169 \\x^2 &= 13^2 \\x &= 13\end{aligned}$$

Ans. $x = 13$

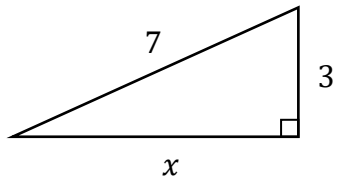
(3)



$$\begin{aligned}x^2 &= 2^2 + \sqrt{21}^2 \\&= 25 \\x^2 &= 5^2 \\x &= 5\end{aligned}$$

Ans. $x = 5$

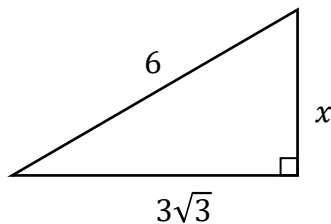
(4)



$$\begin{aligned}x^2 &= 7^2 - 3^2 = 40 \\x &= \sqrt{40} = 2\sqrt{10}\end{aligned}$$

Ans. $x = 2\sqrt{10}$

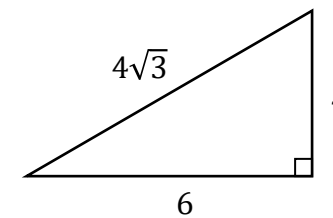
(5)



$$\begin{aligned}x^2 &= 6^2 - (3\sqrt{3})^2 \\x^2 &= 9 = 3^2 \\x &= 3\end{aligned}$$

Ans. $x = 3$

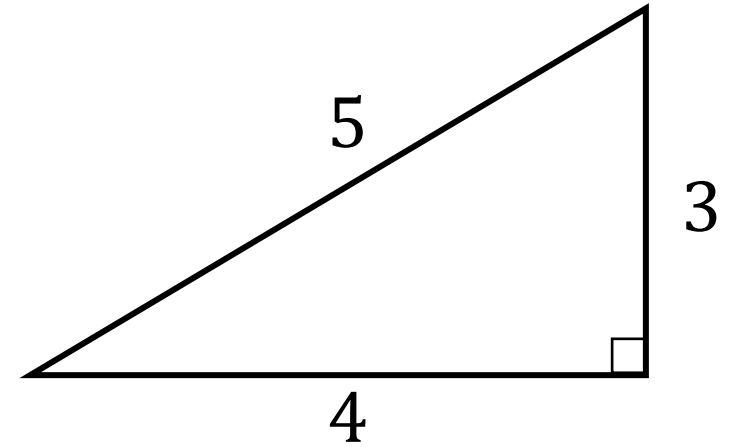
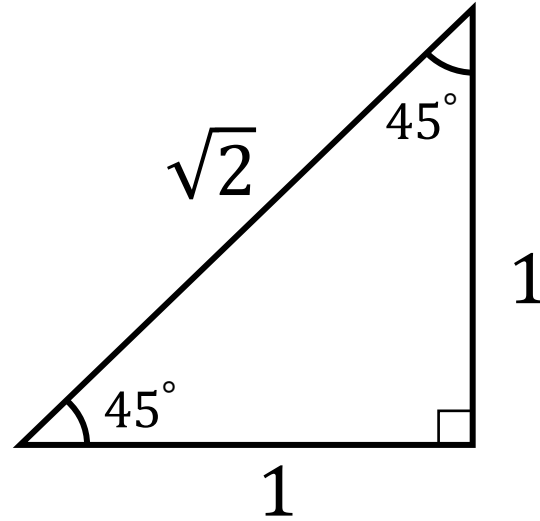
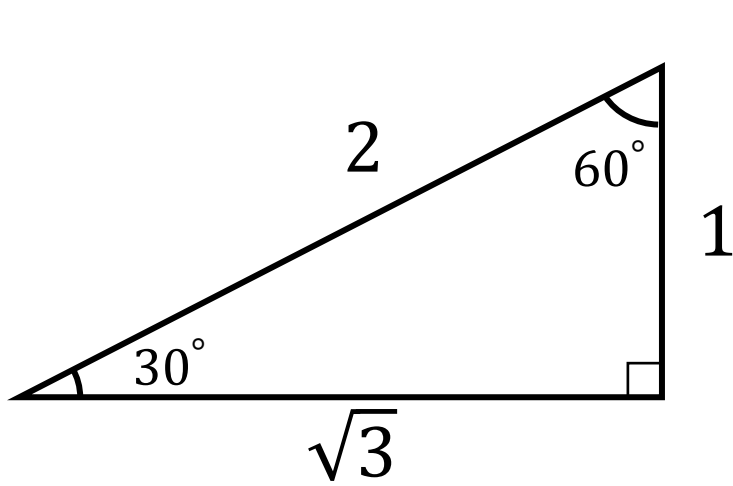
(6)



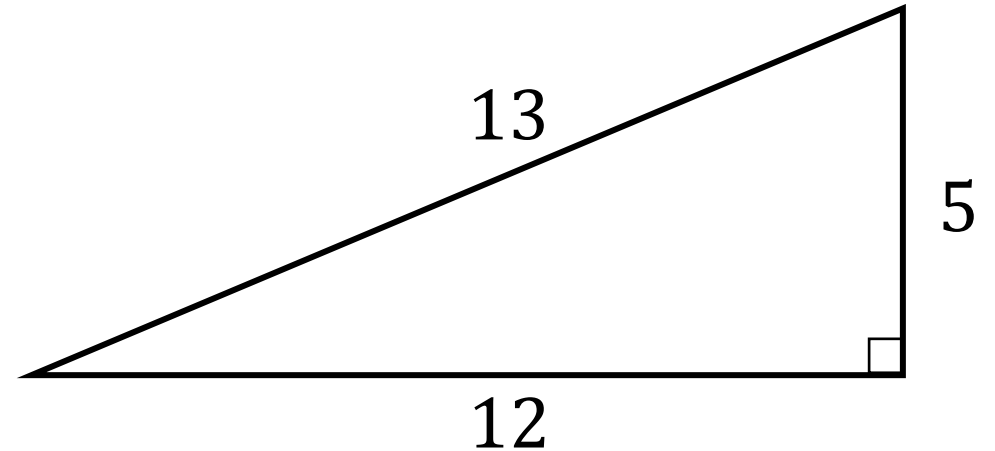
$$\begin{aligned}x^2 &= (4\sqrt{3})^2 - 6^2 \\x^2 &= 48 - 36 = 12 \\x &= \sqrt{12} = 2\sqrt{3}\end{aligned}$$

Ans. $x = 2\sqrt{3}$

特徴的な直角三角形

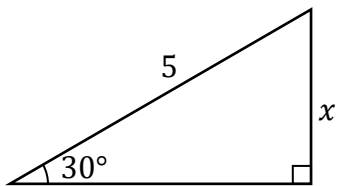


この4つの三角形の角度と辺の長さの比は全て覚えること!



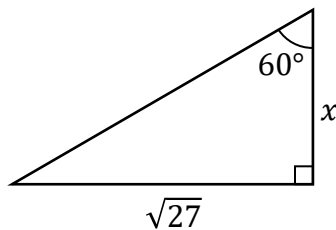
練習問題5

(1)



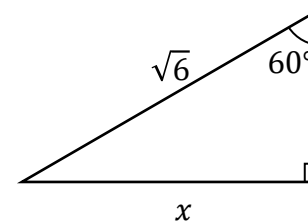
Ans. $x =$ _____

(2)



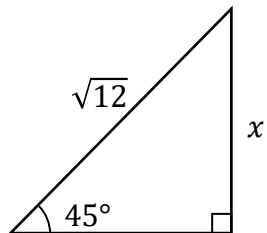
Ans. $x =$ _____

(3)



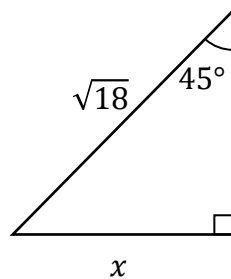
Ans. $x =$ _____

(4)



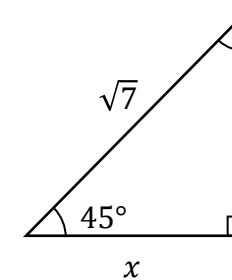
Ans. $x =$ _____

(5)



Ans. $x =$ _____

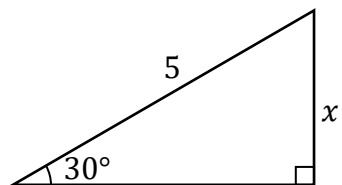
(6)



Ans. $x =$ _____

練習問題5 (解答)

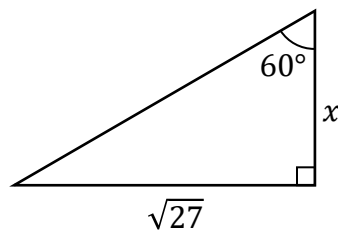
(1)



$$\begin{aligned} 1 : 2 &= x : 5 \\ 2x &= 5 \\ x &= \frac{5}{2} \end{aligned}$$

Ans. $x = \frac{5}{2}$

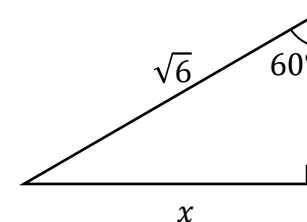
(2)



$$\begin{aligned} 1 : \sqrt{3} &= x : \sqrt{27} \\ \sqrt{3}x &= \sqrt{27} = 3\sqrt{3} \\ x &= 3 \end{aligned}$$

Ans. $x = 3$

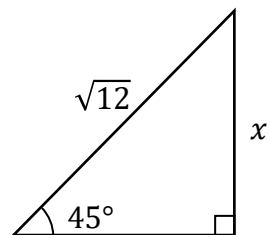
(3)



$$\begin{aligned} \sqrt{3} : 2 &= x : \sqrt{6} \\ 2x &= \sqrt{18} = 3\sqrt{2} \\ x &= \frac{3}{2}\sqrt{2} \end{aligned}$$

Ans. $x = \frac{3}{2}\sqrt{2}$

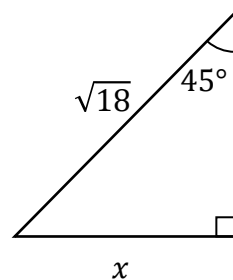
(4)



$$\begin{aligned} 1 : \sqrt{2} &= x : \sqrt{12} \\ \sqrt{2}x &= \sqrt{12} \\ x &= \sqrt{6} \end{aligned}$$

Ans. $x = \sqrt{6}$

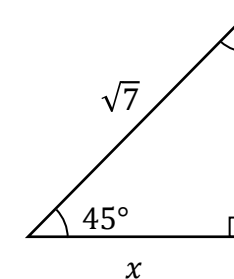
(5)



$$\begin{aligned} 1 : \sqrt{2} &= x : \sqrt{18} \\ \sqrt{2}x &= \sqrt{18} = 3\sqrt{2} \\ x &= 3 \end{aligned}$$

Ans. $x = 3$

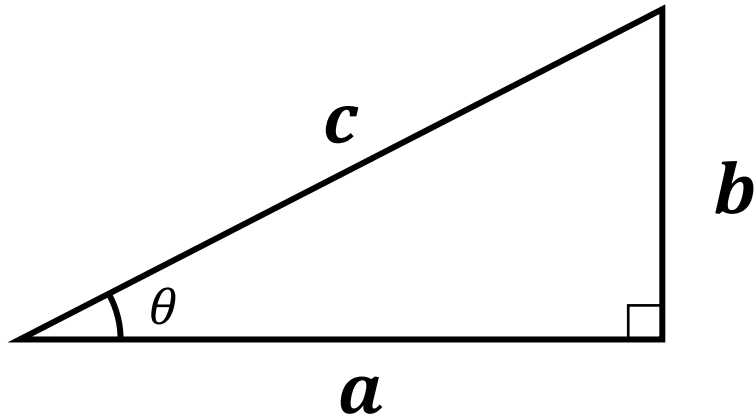
(6)



$$\begin{aligned} 1 : \sqrt{2} &= x : \sqrt{7} \\ \sqrt{2}x &= \sqrt{7} \\ x &= \frac{\sqrt{7}}{\sqrt{2}} = \frac{\sqrt{14}}{2} \end{aligned}$$

Ans. $x = \frac{\sqrt{14}}{2}$

直角三角形と三角比



$$\sin \theta = \frac{b}{c} = \frac{b}{\sqrt{a^2 + b^2}}$$

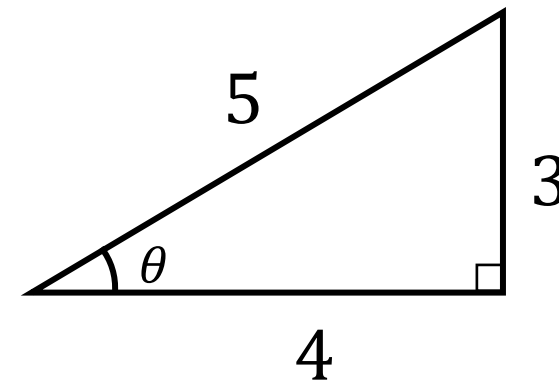
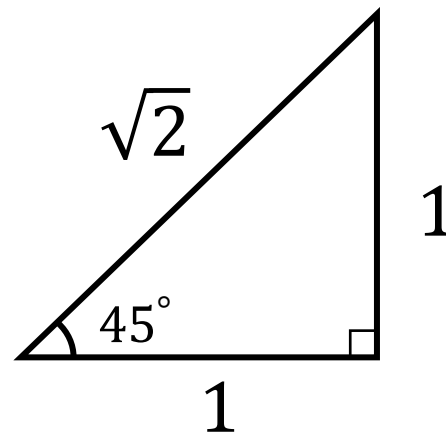
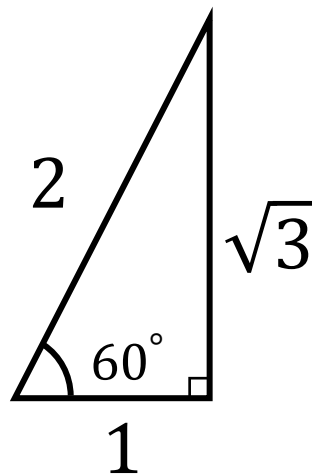
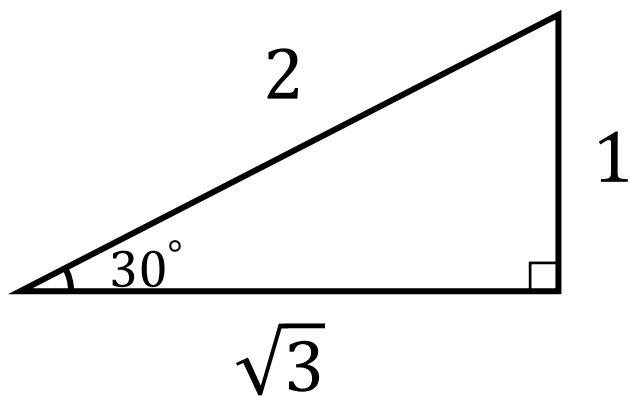
$$\cos \theta = \frac{a}{c} = \frac{a}{\sqrt{a^2 + b^2}}$$

$$\tan \theta = \frac{b}{a}$$

三角比は、直角三角形の
2辺の長さの比を表したもの

その比は角度 θ によって変化するが、
角度 θ が分からなければ値が導出できないわけではない

三角形と三角比



$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

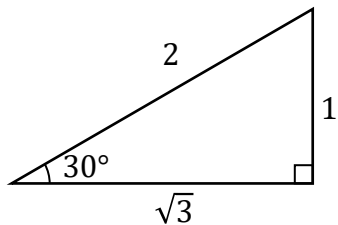
$$\tan 45^\circ = 1$$

$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

$$\tan \theta = \frac{3}{4}$$

練習問題6



(1) $\sin 60^\circ$

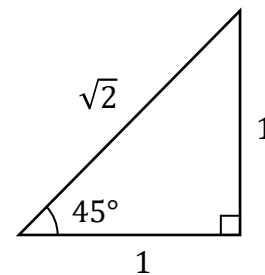
Ans. _____

(2) $\cos 60^\circ$

Ans. _____

(3) $\tan 60^\circ$

Ans. _____



(4) $\sin 45^\circ$

Ans. _____

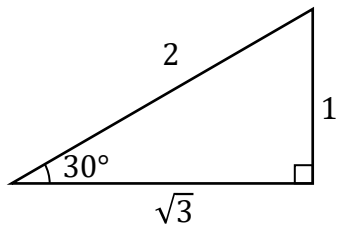
(5) $\cos 45^\circ$

Ans. _____

(6) $\tan 45^\circ$

Ans. _____

練習問題6 (解答)



(1) $\sin 60^\circ$

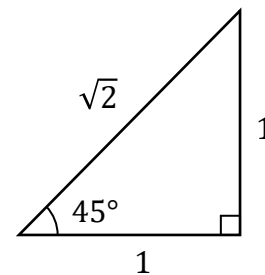
Ans. $\frac{\sqrt{3}}{2}$

(2) $\cos 60^\circ$

Ans. $\frac{1}{2}$

(3) $\tan 60^\circ$

Ans. $\sqrt{3}$



(4) $\sin 45^\circ$

Ans. $\frac{1}{\sqrt{2}}$

(5) $\cos 45^\circ$

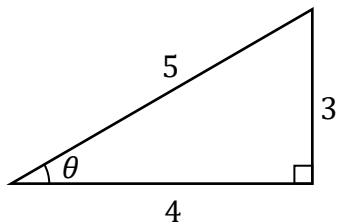
Ans. $\frac{1}{\sqrt{2}}$

(6) $\tan 45^\circ$

Ans. 1

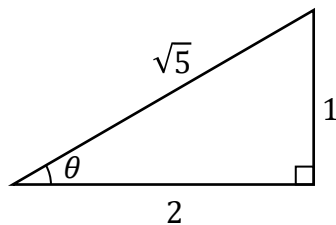
練習問題7

(1) $\sin \theta$



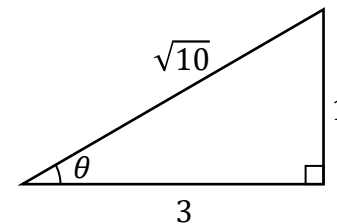
Ans. _____

(2) $\cos \theta$



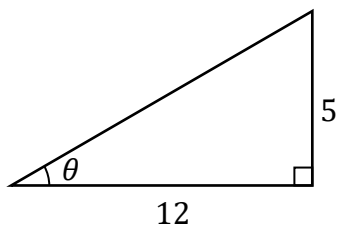
Ans. _____

(3) $\tan \theta$



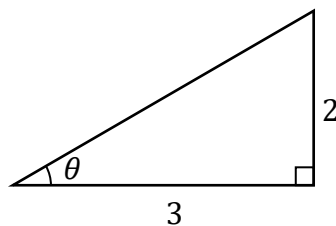
Ans. _____

(4) $\sin \theta$



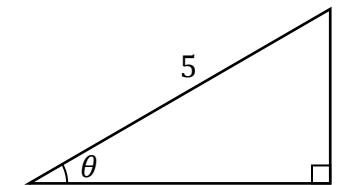
Ans. _____

(5) $\cos \theta$



Ans. _____

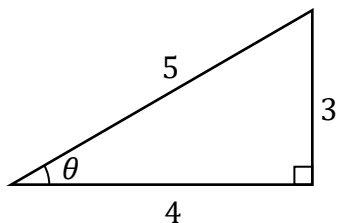
(6) $\tan \theta$



Ans. _____

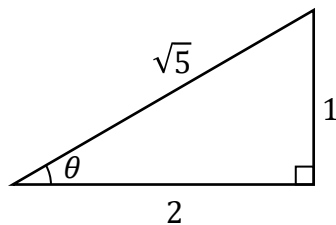
練習問題7 (解答)

(1) $\sin \theta$



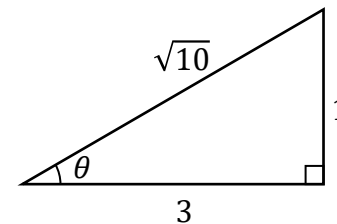
Ans. $\frac{3}{5}$

(2) $\cos \theta$



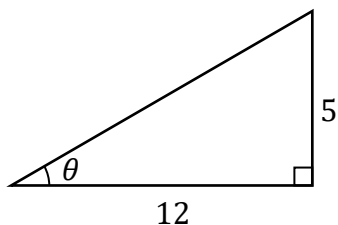
Ans. $\frac{2}{\sqrt{5}}$

(3) $\tan \theta$



Ans. $\frac{1}{3}$

(4) $\sin \theta$

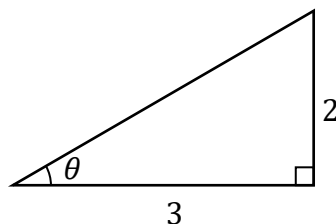


$$\sin \theta = \frac{5}{\sqrt{12^2 + 5^2}}$$

$$= \frac{5}{\sqrt{169}} = \frac{5}{13}$$

Ans. $\frac{5}{13}$

(5) $\cos \theta$



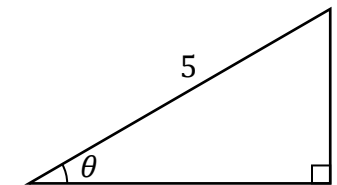
$$\cos \theta = \frac{3}{\sqrt{3^2 + 2^2}}$$

$$= \frac{3}{\sqrt{13}}$$

$$= \frac{3\sqrt{13}}{13}$$

Ans. $\frac{3\sqrt{13}}{13}$

(6) $\tan \theta$



$$\tan \theta = \frac{2}{\sqrt{5^2 - 2^2}}$$

$$= \frac{2}{\sqrt{21}}$$

$$= \frac{2\sqrt{21}}{21}$$

Ans. $\frac{2\sqrt{21}}{21}$

ご聴講ありがとうございました
ございました!!